Calculating Survival Probabilities

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I Introduction

A recent discussion on AAEFE-L, an e-mail list server for members of the American Academy of Economic and Financial Experts, revealed that a concise explanation of how to calculate the survival probabilities used in the Life/Participation/Employment (LPE) method is absent from the forensic economics literature.[1] This note remedies this situation by presenting an overview of the columns found in a life table, followed examples of survival probability calculations for both whole and fractional ages.

II Life Table Overview[2]

Table 1 presents an abbreviated life table for all females, regardless of race. This abbreviated table is taken from Table 3 of *United States Life Tables, 2004*, published by the National Center for Health Statistics (NCHS). The first column in Table 1 of this paper specifies the age ranges for each row; these are one-year ranges except for the last, which corresponds to ages of 100 or more.

This life table is a period life table: Rather than being based on the experience of a cohort of individuals born in the same year, a period table presents what would happen to a synthetic cohort through time if it experienced the death rates specified in column (2). These death rates are written as q(x), and equal the probability of dying between ages x and x+1, for integer values of x from 0 to 99. The probabilities are based on death registrations in the U.S. for ages 0 to 84; for ages 85 to 99, the probabilities are based on Medicare death rates. In the last row of the table, q(100) equals the probability of dying for ages greater than 100. This probability is set equal to one.

The q(x) and an initial synthetic cohort of 100,000 persons at age 0 determine the rest of the table.[3] Column (3) shows $\ell(x)$, the number of persons surviving from the initial cohort to age x, and column (4) shows d(x), the number of persons dying in each age range. For $x \geq 1$, $\ell(x)$ equals $\ell(x-1) - d(x-1)$. The number of deaths is specified by the q(x) and $\ell(x)$: $d(x) = q(x) \cdot \ell(x)$.

The number of person-years lived between ages x and x+1 is shown in column (5) and is written as L(x). For $1 \leq x \leq 99$, $L(x) = \ell(x) - d(x)/2$. For x=0, L(x) reflects an adjustment which accounts for the number of deaths in a year occurring for infants born in the previous year. L(100) is the result of an iterative process that continues until the number of person-years lived is essentially zero; this occurs somewhere between ages 110 and 120. Note that in the table, L(100) is the number of person-years lived from age 100 to infinity. Column (6) is the total number of person-years lived above age x, and is written as T(x). It is the sum of L(x) in column (5), starting at x and continuing through the end of the table. The last column of the table, column (7), is the remaining life expectancy at age x. It is represented by e(x) and equals $T(x)/\ell(x)$.

III Calculating Survival Probability – Whole Ages

If the dates of the loss periods correspond to exact ages, the required survival probabilities are easily calculated from the $\ell(x)$. For example, for a female, if the loss starts exactly at age 20, the probability of surviving one additional year is 0.999545. This probability is calculated in Table 2, and equals $\ell(21)/\ell(20)$ or 98,899 divided by 98,944.[4] This is the probability that would be used to reduce the first year of loss in a wrongful death case.[5] The fourth year of loss would be reduced by multiplying by 0.998130, or $\ell(24)$ divided by $\ell(20)$.

Most economic damage reports divide losses between past and future periods, with the scheduled trial date serving as the point of demarcation between the past and the future. This presents a choice to the forensic economist in personal injury cases: Whether to calculate all survival probabilities as of the date of the injury or as of the trial date, setting the probability of survival equal to one for the past period. The first option will understate the loss estimates, since losses are reduced for the probability of an event that, if the case goes forward, did not happen. The second option recognizes the known, or assumed, circumstances as of the date of the trial. If the loss estimates are calculated using the LPE method, and if the probabilities of labor force participation and employment are set equal to their expected value as of the date of the injury, the second option will neither understate or overstate the loss estimates.[6]

Under the second approach in a personal injury case, it is assumed that the plaintiff survives up until the start of the future loss period. Accordingly, the survival probability for past losses should be set equal to one, and the survival probabilities corresponding to each future loss should be based on the probability of surviving from the start of the future loss period. For example, suppose the plaintiff was a female injured on her 20th birthday, but that because of trial delays the future loss period doesn't start until her 26th birthday. The survival probability associated with the eighth future loss is 0.995013, or $\ell(34)$ divided by $\ell(26)$.

IV Calculating Survival Probability – Fractional Ages[7]

Of course, it would be an extreme coincidence if the start of the past and future loss periods corresponded exactly to a plaintiff's or decedent's birthday.[8] More often than not, survival probabilities for a fractional age will need to be calculated. These calculations must be based on some assumption about the functional form the survival curve determined by $\ell(x)$ takes between integer values of x. It is common to assume either (1) a linear, (2) an exponential, or (3) a hyperbolic function. Each of these assumptions results in an equation for $\ell(x+t)$, or for some function of $\ell(x+t)$, that is a weighted average of

$\ell(x)$ and $\ell(x+1)$, or of expressions involving $\ell(x)$ and $\ell(x+t)$, with the weights being determined by the value of t (for $0 \le t \le 1$).

The linear function assumption is equivalent to assuming that deaths between age x and x+1 are uniformly distributed, so that, for $0 \le t \le 1$, $\ell(x+t)$ decreases linearly until reaching $\ell(x+1)$. The slope of the line connecting $\ell(x)$ and $\ell(x+1)$ equals $\ell(x+1) - \ell(x)$, so that

$$\begin{aligned}\ell(x+t) &= \ell(x) + t\cdot[\,\ell(x+1) - \ell(x)] \\ &= t\cdot \ell(x+1) + (1-t)\cdot \ell(x) \qquad [1]\end{aligned}$$

The exponential function assumption is also known as the constant force of mortality assumption, and assumes that $\ell(x+t) = ab^t$. Under this assumption, $\ell(x) = ab^0$ and $\ell(x+1) = ab^1$. From this it follows that $a = \ell(x)$ and $b = \ell(x+1)/\ell(x)$, and that

$$\ell(x+t) = \ell(x)\cdot[\,\ell(x+1)/\ell(x)]^t$$

$$\ln(\ell(x+t)) = \ln(\ell(x)) + t\cdot[\ln(\ell(x+1) - \ln(\ell(x))]$$

$$\ln(\ell(x+t)) = t\cdot\ln(\ell(x+1)) + (1 - t)\cdot\ln(\ell(x)) \qquad [2]$$

The hyperbolic function assumption (also known as Balducci's assumption) assumes $\ell(x+t) = 1/[a + b\cdot t]$ or, equivalently, $1/\ell(x+t) = a + b\cdot t$. Setting t equal to 0 and 1 in the latter expression produces $1/\ell(x) = a$ and $1/\ell(x+1) = a + b$. From this it follows that $b=1/\ell(x+1) - 1/\ell(x)$ and that

$$1/\ell(x+t) = 1/\ell(x) + t\cdot[1/\ell(x+1) - 1/\ell(x)]$$

$$1/\ell(x+t) = t/\ell(x+1) + (1 - t)/\ell(x) \qquad [3]$$

Equations [1], [2] and [3] all express $\ell(x+t)$ (or some form of $\ell(x+t)$) as a weighted average of some form of $\ell(x)$ and $\ell(x+1)$, with the weights determined by the value of t. Table 3 shows the calculation of the survival probabilities that would be used in calculating the economic losses of a female plaintiff born on July 4, 1960 and injured on November 1, 2008, using each of these equations. In Table 3, it is assumed that the loss period starts on the day of the plaintiff's injury and that the future loss period starts on February 1, 2010, the date of the trial. Accordingly, the past loss period runs from November 1, 2008 through January 31, 2010. In keeping with the second option discussed above, the survival probabilities for this period are assumed to equal one. If losses in future years are presented as ending at the end of each calendar year, then the required survival probabilities correspond to the probability of surviving from January 31, 2010 to December 31, 2010; to December 31, 2011; to December 31, 2012; and so on.

The first ten columns in Table 3 are common to all three assumptions, with the first column showing the dates the injured plaintiff survives to. The date in the first row is the end of the past loss period, and survival probabilities are calculated from this date to the date found in each of the subsequent rows. The second column shows the age of the plaintiff as of the date in the first column. Column (3), "Date for $\ell(x)$", is the birthday prior to the date in the first column – the first row corresponds to the plaintiff's 49th birthday, the second row corresponds to her 50th birthday, and so on. Column (4), "$\ell(x)$", is the number of persons in the synthetic cohort living to the exact age 49, age 50, and so on. Similarly, column (5) corresponds to the plaintiff's birthday immediately following the date in the first column. These birthdays correspond to age 50, age 51, age 52, and so on. The entries in column (6) correspond to the number of persons in the synthetic cohort living to the exact age 50, age 51, age 52, and so on.

The "Days1" column contains the number of days from "Date for $\ell(x)$" until "Date". Similarly, the "Days2" column contains the number of days from "Date" until "Date for $\ell(x+1)$". That is, "Days1" corresponds to the interval starting with the birthday prior to "Date" and ending with "Date", while "Days2" corresponds to the interval from "Date" until the next birthday. Their sum is shown in column (9) and equals 365 except when the period includes February 29th in a leap year. These values determine the value of "t" in the equations above: t equals Days1 divided by the total number of days and is shown in column (10).

Columns (11) and (12) show, respectively, the interpolated values of $\ell(x)$ corresponding to "Date", and the probabilities of surviving from the end of the past loss period until "Date" for the uniform distribution assumption. For example, consider the probability of surviving from January 31, 2010 until year-end 2013. The first step is to calculate $\ell(49.6)$, the value of $\ell(x)$ corresponding to January 31, 2010. The birthdays immediately prior and after this point in time are July 4, 2009 and July 4, 2010. These are the dates for $\ell(49)$ and $\ell(50)$, respectively. The value for $\ell(49)$ is taken directly from the NCHS life table for all females and equals 95,733. Similarly, $\ell(50)$ equals 95,445. There are 211 days from July 4, 2009 until January 31, 2010, and 154 days until the plaintiff's next birthday. The total number of days equals 365, and t equals 0.57808 (211 divided by 365). Assuming that deaths between integer ages are uniformly distributed, the interpolated value of $\ell(49.6)$ equals $0.57808 \cdot 95{,}445 + (1 - 0.57808) \cdot 95{,}733$, or 95,567.

The second step is to calculate $\ell(53.5)$, the value of $\ell(x)$ for December 31, 2013.   The birthdays immediately before and after year-end 2013 are July 4, 2013 and July 4, 2014. These are the dates for $\ell(53)$ and $\ell(54)$, respectively. The value for $\ell(53)$ equals 94,462, and $\ell(54)$ equals 94,085; these values come directly from the NCHS life table for all females. There are 180 days from July 4, 2013 until year-end 2013, and 185 days from year-end 2013 until the plaintiff's next birthday. The total number of days equals 365, and t equals 0.49315 (180 divided by 365). One minus t equals 0.50685. Assuming that deaths between integer ages are uniformly distributed, the interpolated value of $\ell(53.5)$ equals 94,276 $(0.49315 \cdot 94{,}085 + 0.50685 \cdot 94{,}462)$. The probability of surviving from January 31, 2010 until December 31, 2013 is 0.98649 (94,276 divided by 95,567).

The next two columns, numbered (13) and (14), show the interpolated values of $\ell(x)$ and the corresponding survival probabilities for the constant force of mortality assumption. Consider again the probability of surviving from January 31, 2010 until year-end 2013. The interpolated value of $\ell(49.6)$ is calculated using equation [2] above:

$$\ln(\ell(49.6)) = 0.57808 \cdot \ln(95{,}445) + (1 - 0.57808) \cdot \ln(95{,}733)$$

$$= 11.467577$$

This gives $e^{11.467577}$ or 95,566 for $\ell(49.6)$. The value of $\ell(53.5)$ is given by

$$\ln(\ell(53.5)) = 0.49315 \cdot \ln(94{,}085) + 0.50685 \cdot \ln(94{,}462)$$

$$= 11.453981$$

This gives $e^{11.453981}$ or 94,276 for $\ell(53.5)$. The probability of surviving from January 31, 2010 until December 31, 2013 is 0.98650 (94,276 divided by 95,566).

The interpolated values of $\ell(x)$ and survival probabilities corresponding to the Balducci assumption are shown in columns (15) and (16). The interpolated value of $\ell(x)$ corresponding to January 31, 2010 is calculated using equation [3] above:

$$1/[\ell(49.6)] = 0.57808/95{,}445 + (1 - 0.57808)/95{,}733$$

$$= 1.046394 \cdot 10^{-5}$$

The interpolated value for $\ell(49.6)$ equals the inverse of $1.046394 \cdot 10^{-5}$ or 95,566, the same value as obtained under the constant force of mortality assumption. The value of $\ell(x)$ corresponding to year-end 2013 is given by

$$1/[\ell(53.5)] = 0.49315/94{,}085 + 0.50685/94{,}462$$

$$= 1.060719 \cdot 10^{-5}$$

The interpolated value for $\ell(53.5)$ equals the inverse of $1.060719 \cdot 10^{-5}$ or 94,276. Again, this is the same value as was obtained under the constant force of mortality assumption. The probability of surviving from January 31, 2010 until December 31, 2013 is 0.98650 (94,276 divided by 95,566). The rightmost column of Table 3 shows the maximum difference between the three interpolated survival probabilities for each row of the table. The largest difference in this example is 0.00134 and the average of the maximum differences in 0.00041.

In order to investigate the magnitude of the differences among the survival probabilities calculated under each of the three assumptions discussed above, the probability of surviving from age 1 through age 99 was calculated by month, where a "month" was interpreted an equal to one-twelfth of a year. For females, the largest difference equaled 0.00129 and the average of the largest differences (for the interpolated ages only) equaled 0.00014. The comparable results for male survival probabilities are 0.00105 and 0.00015.

The resulting interpolated values of the $\ell(x)$ allow one to calculate the survival probability between any two of the ages shown. This was done for all integer starting ages from 1 to 75, through all ending monthly ages to age 99. The resulting maximum differences among the three assumptions for each starting age, along with the means of the maximum difference for only the interpolated ages, are shown in Table 4. For both females and males, the maximum difference is less than 0.0018 and the average of the maximum difference for the interpolated ages is less than 0.00089. These are not differences that are likely to significantly impact calculations of economic losses.

V Rounding Values of $\ell(x)$

All of the calculations presented in this paper rounded the $\ell(x)$ before the survival probabilities were calculated in order that the results presented could be duplicated with a hand calculator. Since the NCHS makes Excel versions of its life tables available containing non-integer values for $\ell(x)$, and for d(x), L(x) and T(x) as well, it is natural to ask if rounding such values makes a significant difference. To investigate this question, the life tables for all females and all males were duplicated, using rounded values of ℓ(x), d(x), L(x) and T(x) in each row. For females, the difference in the resulting ℓ(x) was no more than 2 persons, corresponding to ages where the number of survivors in the synthetic cohort exceeded 82,000 persons. For males, the difference in the resulting ℓ(x) was no more than 4 persons. These differences corresponded to ages where the number of cohort survivors exceeded 94,000 persons.

VI Conclusion

The examples presented above show that calculating survival probabilities for whole or fractional ages is relatively straightforward. Moreover, there does not seem to be a significant difference among the results obtained for interpolated ages using the three common assumptions concerning the shape of the survivor curve between integer ages. In this regard, it is worth noting that the assumption of uniformly distributed deaths between exact ages x and x+1 is equivalent to the assumption made in Arias (2007) that ℓ(x) declines linearly between x and x+1 for ages 1 to 99. This serves as a basis for a small preference for the uniformly distributed assumption over the other two.

Finally, using rounded values of ℓ(x), d(x), L(x) and T(x) will result in some minor differences in the derived values of ℓ(x) from the published tables. Given that Excel versions with the unrounded values are readily available, loss calculations should probably not utilize rounded values. However, doing so is not likely to result in materially different results.

Table 1: Abbreviated Life Table for All Females, United States, 2004

(1) Age	(2) Probability of dying between ages x to x+1 q(x)	(3) Number surviving to age x ℓ(x)	(4) Number dying between ages x to x+1 d(x)	(5) Person-years lived between ages x to x+1 L(x)	(6) Total number of person-years lived above age x T(x)	(7) Expectation of life at age x e(x)
0-1	0.006091	100,000	609	99,465	8,038,173	80.4
1-2	0.000457	99,391	45	99,368	7,938,708	79.9
2-3	0.000267	99,346	26	99,332	7,839,340	78.9
3-4	0.000197	99,319	20	99,309	7,740,008	77.9
:	:	:	:	:	:	:
:	:	:	:	:	:	:
98-99	0.242924	6,194	1,505	5,442	18,452	3.0
99-100	0.262224	4,689	1,230	4,075	13,010	2.8
100 or over	1.000000	3,460	3,460	8,935	8,935	2.6

Table 2: Survival Probability for All Females by Start of Loss Period

Age That Loss Occurs	ℓ(x) = Number Surviving to Age x	Starting Age								
		18	19	20	21	22	23	24	25	26
18	99,033	1.000000								
19	98,989	0.999556	1.000000							
20	98,944	0.999101	0.999545	1.000000						
21	98,899	0.998647	0.999091	0.999545	1.000000					
22	98,853	0.998182	0.998626	0.999080	0.999535	1.000000				
23	98,807	0.997718	0.998161	0.998615	0.999070	0.999535	1.000000			
24	98,759	0.997233	0.997677	0.998130	0.998584	0.999049	0.999514	1.000000		
25	98,710	0.996738	0.997182	0.997635	0.998089	0.998553	0.999018	0.999504	1.000000	
26	98,661	0.996244	0.996687	0.997140	0.997594	0.998058	0.998522	0.999008	0.999504	1.000000
27	98,609	0.995719	0.996161	0.996614	0.997068	0.997532	0.997996	0.998481	0.998977	0.999473
28	98,556	0.995183	0.995626	0.996079	0.996532	0.996996	0.997460	0.997944	0.998440	0.998936
29	98,500	0.994618	0.995060	0.995513	0.995966	0.996429	0.996893	0.997377	0.997873	0.998368
30	98,442	0.994032	0.994474	0.994926	0.995379	0.995842	0.996306	0.996790	0.997285	0.997780
31	98,380	0.993406	0.993848	0.994300	0.994752	0.995215	0.995678	0.996162	0.996657	0.997152
32	98,314	0.992740	0.993181	0.993633	0.994085	0.994547	0.995010	0.995494	0.995988	0.996483
33	98,244	0.992033	0.992474	0.992925	0.993377	0.993839	0.994302	0.994785	0.995279	0.995773
34	98,169	0.991276	0.991716	0.992167	0.992619	0.993081	0.993543	0.994026	0.994519	0.995013
35	98,088	0.990458	0.990898	0.991349	0.991800	0.992261	0.992723	0.993206	0.993699	0.994192

Equals 98,899 divided by 98,944 or ℓ(21) divided by ℓ(20).

Equals 98,169 divided by 98,661 or ℓ(34) divided by ℓ(26).

Table 3: Calculation of Survival Probability from 1/31/2010 to year-end through age 99.5

Date of Birth is: 7/4/1960

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Date	**Age**	**Date for ℓ(x)**	**ℓ(x)**	**Date for ℓ(x+1)**	**ℓ(x+1)**	**Days1**	**Days2**	**Total Days**	**t**
1/31/2010	49.6	7/4/2009	95,733	7/4/2010	95,445	211	154	365	0.57808
12/31/2010	50.5	7/4/2010	95,445	7/4/2011	95,139	180	185	365	0.49315
12/31/2011	51.5	7/4/2011	95,139	7/4/2012	94,813	180	186	366	0.49180
12/31/2012	52.5	7/4/2012	94,813	7/4/2013	94,462	180	185	365	0.49315
12/31/2013	53.5	7/4/2013	94,462	7/4/2014	94,085	180	185	365	0.49315
:	:	:	:	:	:	:	:	:	:
:	:	:	:	:	:	:	:	:	:
12/31/2057	97.5	7/4/2057	7,988	7/4/2058	6,194	180	185	365	0.49315
12/31/2058	98.5	7/4/2058	6,194	7/4/2059	4,689	180	185	365	0.49315
12/31/2059	99.5	7/4/2059	4,689	7/4/2060	3,460	180	186	366	0.49180

Table 3 (Continued)

(11)	(12)	(13)	(14)	(15)	(16)	(17)
** Uniform Distribution **		*** Constant Force of Mortality ***		** Balducci Assumption **		
Interpolated ℓ(x) for "Date"	Probability of Surviving from 1/31/2010	Interpolated ℓ(x) for "Date"	Probability of Surviving from 1/31/2010	Interpolated ℓ(x) for "Date"	Probability of Surviving from 1/31/2010	Maximum Absolute Difference in Survival Probability
95,567	1.00000	95,566	1.00000	95,566	1.00000	0.00000
95,294	0.99714	95,294	0.99715	95,294	0.99715	0.00001
94,979	0.99385	94,979	0.99386	94,978	0.99385	0.00001
94,640	0.99030	94,640	0.99031	94,640	0.99031	0.00001
94,276	0.98649	94,276	0.98650	94,276	0.98650	0.00001
:	:	:	:	:	:	:
:	:	:	:	:	:	:
7,103	0.07432	7,046	0.07373	6,990	0.07314	0.00118
5,452	0.05705	5,400	0.05651	5,348	0.05596	0.00109
4,085	0.04274	4,038	0.04225	3,992	0.04177	0.00097
					Largest Maximum Difference:	0.00134
					Mean Maximum Difference:	0.00041

Table 4: Differences in Interpolated Results for Survival Probabilities Through Age 99

	Females		Males	
Starting Age	Largest Maximum Difference	Mean of Maximum Difference	Largest Maximum Difference	Mean of Maximum Difference
1	0.0012878	0.0001441	0.0010478	0.0001457
2	0.0012884	0.0001456	0.0010484	0.0001473
3	0.0012888	0.0001471	0.0010487	0.0001489
4	0.0012890	0.0001487	0.0010490	0.0001505
5	0.0012892	0.0001503	0.0010492	0.0001521
6	0.0012894	0.0001519	0.0010494	0.0001537
7	0.0012896	0.0001536	0.0010496	0.0001554
8	0.0012898	0.0001553	0.0010498	0.0001571
9	0.0012899	0.0001570	0.0010499	0.0001588
10	0.0012901	0.0001588	0.0010500	0.0001606
11	0.0012902	0.0001606	0.0010502	0.0001625
12	0.0012904	0.0001625	0.0010503	0.0001643
13	0.0012905	0.0001644	0.0010504	0.0001663
14	0.0012908	0.0001663	0.0010507	0.0001683
15	0.0012911	0.0001683	0.0010512	0.0001703
16	0.0012915	0.0001703	0.0010518	0.0001724
17	0.0012920	0.0001724	0.0010526	0.0001747
18	0.0012925	0.0001746	0.0010536	0.0001770
19	0.0012931	0.0001768	0.0010548	0.0001793
20	0.0012937	0.0001791	0.0010560	0.0001818
21	0.0012942	0.0001814	0.0010573	0.0001844
22	0.0012949	0.0001839	0.0010588	0.0001870
23	0.0012955	0.0001863	0.0010603	0.0001897
24	0.0012961	0.0001889	0.0010618	0.0001925
25	0.0012967	0.0001916	0.0010633	0.0001953
26	0.0012974	0.0001943	0.0010648	0.0001983
27	0.0012981	0.0001971	0.0010662	0.0002013
28	0.0012988	0.0001999	0.0010677	0.0002044
29	0.0012995	0.0002029	0.0010691	0.0002076
30	0.0013003	0.0002059	0.0010705	0.0002108
31	0.0013011	0.0002091	0.0010720	0.0002142
32	0.0013020	0.0002123	0.0010734	0.0002177
33	0.0013029	0.0002156	0.0010750	0.0002213
34	0.0013039	0.0002190	0.0010765	0.0002250
35	0.0013050	0.0002226	0.0010782	0.0002288
36	0.0013061	0.0002263	0.0010800	0.0002328
37	0.0013074	0.0002302	0.0010819	0.0002369
38	0.0013088	0.0002341	0.0010840	0.0002412

Table 4: Differences in Interpolated Results for Survival Probabilities Through Age 99 (continued)

	Females		Males	
Starting Age	Largest Maximum Difference	Mean of Maximum Difference	Largest Maximum Difference	Mean of Maximum Difference
39	0.0013103	0.0002383	0.0010863	0.0002457
40	0.0013121	0.0002426	0.0010887	0.0002504
41	0.0013140	0.0002471	0.0010914	0.0002553
42	0.0013161	0.0002518	0.0010942	0.0002605
43	0.0013184	0.0002567	0.0010974	0.0002658
44	0.0013208	0.0002618	0.0011008	0.0002715
45	0.0013236	0.0002672	0.0011046	0.0002774
46	0.0013265	0.0002728	0.0011087	0.0002836
47	0.0013298	0.0002788	0.0011132	0.0002902
48	0.0013333	0.0002849	0.0011182	0.0002971
49	0.0013371	0.0002914	0.0011236	0.0003044
50	0.0013411	0.0002983	0.0011295	0.0003121
51	0.0013454	0.0003053	0.0011359	0.0003203
52	0.0013500	0.0003128	0.0011429	0.0003289
53	0.0013550	0.0003207	0.0011505	0.0003381
54	0.0013605	0.0003291	0.0011586	0.0003478
55	0.0013664	0.0003380	0.0011674	0.0003582
56	0.0013729	0.0003474	0.0011767	0.0003693
57	0.0013800	0.0003574	0.0011868	0.0003810
58	0.0013878	0.0003680	0.0011976	0.0003934
59	0.0013963	0.0003794	0.0012095	0.0004068
60	0.0014057	0.0003915	0.0012226	0.0004212
61	0.0014162	0.0004045	0.0012371	0.0004368
62	0.0014278	0.0004184	0.0012533	0.0004536
63	0.0014408	0.0004336	0.0012713	0.0004719
64	0.0014550	0.0004498	0.0012912	0.0004918
65	0.0014705	0.0004674	0.0013129	0.0005132
66	0.0014876	0.0004863	0.0013368	0.0005365
67	0.0015064	0.0005069	0.0013631	0.0005620
68	0.0015273	0.0005294	0.0013923	0.0005897
69	0.0015505	0.0005539	0.0014249	0.0006203
70	0.0015764	0.0005807	0.0014613	0.0006541
71	0.0016052	0.0006103	0.0015020	0.0006913
72	0.0016373	0.0006429	0.0015474	0.0007325
73	0.0016733	0.0006789	0.0015986	0.0007783
74	0.0017138	0.0007190	0.0016569	0.0008294
75	0.0017595	0.0007636	0.0017237	0.0008868

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[1] In addition to the LPE method, survival probabilities can be used by forensic economists to calculate the expected present value of a pension, or to reduce the expected loss to survivors in a death case to reflect their own mortality.

[2] The discussion in this section is based on Arias (2007).

[3] For the first and last age ranges, additional information is required for the number of person-years lived shown in column (5). See Arias (2007) and the discussion below.

[4] The columns after $\ell(x)$ in Table 2 correspond to the age the loss period starts; the rows correspond to the age in the year each subsequent loss occurs. There is nothing special about the ending age of 26 for the columns or of 35 for the rows. The table was stopped at these ages only due to size considerations. Note that $\ell(x)$ in the Excel spreadsheet from the NCHS underlying Table 1 are not integer values even though they are displayed that way. The $\ell(x)$ in Table 2 and all subsequent tables have been rounded to the nearest whole number in order that the calculations can be duplicated using a calculator.

[5] Alternatively, the probability of surviving only six additional months might be used to reduce the first year loss. This approach would be consistent with the logic underlying mid-period discounting for example, though the difference in the overall results would be negligible. Under this approach, the subsequent losses would be reduced using the probabilities of surviving 18 months, 30 months, 42 months, and so on.

[6] Townsend (1997) has addressed this issue in the context of work life expectancy (WLE). He maintains that the pre-injury WLE expectancy should be used, since use of a post-injury WLE may enrich the plaintiff by extending the total loss period. While this is may be true, use of the pre-injury WLE implicitly assumes that the probability of surviving during the past loss period was less than one as of the trial date, contrary to the assumed or known circumstances.

[7] The calculations discussed in this section are based on Bowers, *et. al*. (1986) and on Arcone (2008).

[8] Even if they did, survival probabilities for fractional ages might be needed, for example, if annual loss estimates corresponded to a calendar year.