An Affirmative Argument for Use of the Net Discount Rate

David G. Tucek

Value Economics, LLC
13024 Vinson Court
St. Louis, MO 63043

Tel: 314/434-8633
Cell: 314/440-4925

David.Tucek@valueeconomics.com

Paper Presented at the
2021 Allied Social Science Association's
Annual Meeting
January 3-5, 2021

Chicago, IL

Draft: 11-01-2020
Do Not Quote Without Permission

Abstract

This paper presents an affirmative argument supporting the use of a historical net discount rate (NDR) to calculate the present value of future lost earnings or medical care costs. The argument relies on two axiomatic propositions:

(1) No one knows what the plaintiff's or decedent's future earnings would have been or what the plaintiff's future medical care costs will be; and

(2) The investment return a plaintiff will receive in the future is unknown.

These propositions lead directly to the conclusion that the best anyone can do is estimate the expected present value of a future loss, which is the basis for the affirmative argument supporting the use of a historical NDR. The paper concludes by outlining a means by which the historical NDR may be implemented that is consistent with this argument.

Introduction

Most, though not all, forensic economists fall into one of two groups: (1) those who rely on a historical net discount rate (NDR) to calculate the present value of future lost earnings or medical care costs; and (2) those who rely on average historical or forecasted growth rates to project losses into the future and current interest rates to discount the future losses to the present.[1] This dichotomy has existed almost since forensic economics began to be established as a separate/distinct discipline. For example, Nowak (1991) concluded that a fixed net discount rate cannot be used as a general rule for calculating monetary awards, while Pelaez (1991) held that interest rates, wage growth, and inflation are unit root processes making long-run forecasts difficult and that the ratio of the earnings growth rate to the discount rate – the net discount rate – was stationary. More recently, Baumann and Schap (2014 and 2015) have presented evidence concerning the stationarity of both earnings and medical NDRs, while Krueger (2016) has (prematurely) retired the NDR to the dust bin of failed methodologies. Even though it is almost certain that the disagreement between these two groups will never be resolved, this paper presents an affirmative argument supporting the use of the NDR approach, and outlines a means by which it may be implemented that is consistent with this argument.

Foundational Propositions

The argument for use of the NDR rests on two axiomatic propositions:

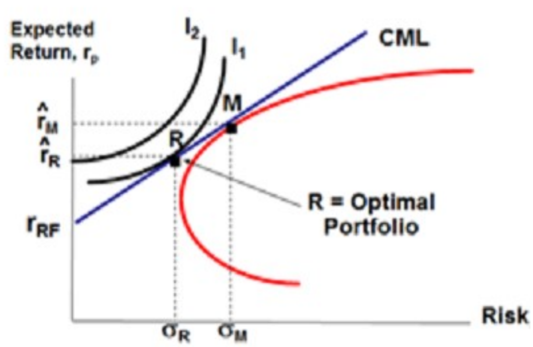
(1) No one knows what the plaintiff's or decedent's future earnings would have been or what the plaintiff's future medical care costs will be; and

(2) The investment return a plaintiff will receive in the future is unknown.

Proposition 1 is clearly self-evident. Future earnings will depend not only on the growth in some base or starting level of earnings but also on the occurrence or avoidance of the risk that the plaintiff might die, not be an active labor force participant even if alive, or that the plaintiff might be unemployed even if active in the labor force. Likewise, future medical costs depend not only on the growth rate used to project those costs, but also on the plaintiff's medical progress and mortality risk.

Proposition 2 is also clearly self-evident. Even if we had perfect foresight concerning the total future returns of all possible investment instruments, we could not know what the plaintiff's future total returns will be since we do not know how any award or settlement amount will be invested initially. Indeed, even if we knew the composition of the initial investment portfolio, we still could not know what the plaintiff's future total returns will be since we do not know how that portfolio will change as time passes. This is easily seen by considering Figure 1, which is a representation plaintiff's portfolio choice decision:

Figure 1 - The Portfolio Choice Decision



[1] Included in the first group are those forensic economists who rely on historical average values of both growth and discount rates since this approach is mathematically equivalent to use of a historical NDR. Likewise, as explained below, the second group includes those forensic economists who rely on both a forecasted growth rate and forecasted interest rates. Finally, not all forensic economists deal with lost future earnings or future medical costs and, consequently, may not fall into either group.

This diagram, which should be familiar to most forensic economists, depicts the basic result of modern portfolio theory that all but the most risk-averse investor will hold a portfolio consisting of both risk-free and risky assets. The vertical axis in this figure measures the expected total return on a portfolio, while the horizontal axis corresponds to the portfolio's risk. The curves labeled I_1 and I_2 represent indifference curves between risk and return for an individual investor. They are upward sloping because it is assumed that the individual is risk-averse – that is, in order to willingly take on more risk, an increase in the expected total return is required. And, like all indifference curves for a given individual, they cannot cross.

Note that a completely risk-averse individual would have a single indifference curve corresponding to the vertical axis, indicating that he will only hold the risk-free asset. Typically, the risk-free asset is described as a near-cash substitute such as short-term Treasury bills. However, it can also be thought of as a portfolio of Treasury securities if "risk" is defined to refer to the risk of default rather than the variation in the total expected portfolio return.

The upward-sloping portion of the red hyperbola is the efficient frontier – it is the set of portfolios of risky assets each with the feature that no other portfolio exists with a higher expected return for a given degree of risk. In the absence of a risk-free asset, the investor will choose a portfolio on the efficient frontier that is just tangent to an indifference curve: all other indifference curves will either be higher and consequently unachievable, or lower, producing less benefit or "utility" to the investor. The point r_{RF} on the vertical axis is the expected return of the risk-free asset. The solid blue line that goes through this point and is tangent to the efficient frontier is known as the Capital Market Line (CML). (This line is also called the Capital Allocation Line or CAL). It shows the return and risk combinations of portfolios made up of the risk-free asset and the market portfolio (point M) of all risky assets. With a risk-free asset, the CML becomes the efficient frontier. Unless an investor is completely risk-averse (*i.e.*, has a return/risk indifference curve lying along the vertical axis), the optimal utility-maximizing portfolio, R, will be some combination of the risk-free asset and the market portfolio of risky assets at point M. And, because r_{RF} and the efficient frontier will shift through time, the combination of the risk-free asset and the market portfolio of risky assets will change through time. Consequently, even if we knew the initial composition of the plaintiff's investment portfolio, we cannot know its composition as time passes and therefore cannot know the total return a plaintiff will receive in the future.

It is clear from the above that both the future losses and the plaintiff's future total returns are random variables. That is, they are variables whose values depend on outcomes of random phenomena. It follows then, that the present value of the future losses is also a random variable and that the best anyone can do is to estimate the expected present value of a future loss – in order to do better than estimating the expected present value of a future loss one would, at a minimum, have to know what the plaintiff's or decedent's future would have been but for the event that gave rise to the tort. Clearly, no one knows this.

The Affirmative Argument

We can state the above conclusion more formally by letting L_i be the future loss in loss year i, and K_i be the corresponding present value factor (a function of the discount rate) constructed in a way such that the present value, P_i, of L_i equals the product $K_i \cdot L_i$. It follows then that:

(1) $E(P_i) = E(K_i \cdot L_i)$ where E( ) denotes the expected value. From this, it follows that:

(2) $E(P_i) = E(K_i) \cdot E(L_i) + Cov(K_i,L_i)$ where Cov( ) denotes the covariance.[2]

[2] For any two random variables, X and Y,

$$\begin{aligned} Cov(X,Y) = E[(X-E[X])\cdot(Y-E[Y])] &= E[X\cdot Y-Y\cdot E[X]-X\cdot E[Y]+E[X]\cdot E[Y]] \\ &= E[X\cdot Y]-E[Y]\cdot E[X]-E[X]\cdot E[Y]+E[X]\cdot E[Y] \\ &= E[X\cdot Y]-E[X]\cdot E[Y] \text{ which gives:} \end{aligned}$$

$E[X\cdot Y]=E[X]\cdot E[Y]+Cov(X,Y)$ or the result relied on in (2) above with X= K_i and Y= L_i.

In other words, even if K_i and L_i equal their expected values, any estimate based on their individual values will be biased downwards if the covariance between K_i and L_i is positive, and biased upwards if it is negative.[3] If L_i is a function of future growth in wages or medical costs, the covariance will be positive for several reasons.

First, consider the Fisher equation:

$$(1+i) = (1+r) \cdot (1+\pi^e)$$

where "i" is the nominal rate of interest; "r" is the real rate of interest; and "π^e" is expected inflation. This shows that expected inflation is the link between nominal interest rates and expected nominal growth rates. Because these two variables are positively correlated, the covariance between K_i and L_i will be positive, and the estimated value of P_i will be biased upwards if it is based on individual values of K_i and L_i and if L_i is a function of future economic growth and inflation.

Second, the real interest rate is determined by the supply of capital (savings) and the demand for capital (investment). Because expected future economic growth is a determinant of both savings and investment, the real rate of interest and expected real growth rates are positively correlated. Once again, if L_i is a function of future growth in wages or medical costs, the covariance between K_i and L_i will be positive and the estimated value of P_i will be biased upwards.

If K_i is expressed on a net discount rate basis, and if the NDR is stationary, L_i will not be a function of future economic growth or inflation and $Cov(K_i,L_i)$ will equal zero. Consequently, use of an NDR eliminates the bias in the estimated value of P_i. Note that it is necessary to test the NDR for stationarity because it is a forecast of the difference between the growth rate and the discount rate. Clearly, if the NDR is not expected to be stable about some mean, but is instead nonstationary, a forecast based on its mean will not be meaningful. Put differently, $E(K_i)$ in (2) above would not be a constant.

An astute reader will note that a given NDR and separate discount and growth rates which produce the same NDR will produce the same results and conclude that adopting the NDR approach adds nothing meaningful to the estimation process. The flaw in this logic is that the separate discount and growth rates need to be consistent – that is, the expectations for economic growth and inflation underlying the chosen discount rate must be consistent with the chosen growth rate. Often, this is clearly not the case, with an extreme example being current interest rates relied on for discounting and with average growth rates calculated over a fifty-year period used to project losses into the future. While this example presents an obvious disconnect between the two variables that determine the NDR which produces identical results as applying the growth and discount rates separately, it is not farfetched since the choice of discount and growth rates are often done in isolation of each other.[4] Again, this need for consistency between the discount and growth rates underlying the NDR translates into a requirement that the NDR be stationary.

Implementing the NDR Approach

Implementing the NDR approach requires completion of three separate tasks: (1) identification of the expected returns the plaintiff will receive in the future; (2) determination of an NDR based on this expected return; and (3) testing the NDR for stationarity. Unlike the conclusion that use of separate growth and discount rates leads to biased estimates of the estimated present value of the plaintiff's future

[3] Of course, such an estimate may founder for other reasons – for example, the underlying forecasted growth rate may be in error or simply unsupported.

[4] Some forensic economists rely on forecasts of both growth and interest rates taken from the same source. While this would seem to meet the requirement of consistency, it is not possible for the forensic economist to reach this conclusion without knowledge of the assumptions that went into the growth and interest rate forecasts and the specification of the underlying model, assuming that such a model exists.

losses, the means of completing these three tasks is not clear cut. The remainder of this paper presents a possible approach to accomplishing each of these three tasks.

The Plaintiff's Expected Future Returns

As noted earlier, while we cannot know what the plaintiff's future total returns will be, we do know that all but the most risk-averse investor will hold a portfolio consisting of both risk-free and risky assets. Consequently, and barring constraints to the contrary, the present value of the future losses should be based on a discount rate that reflects the total returns on such a portfolio.

The key phrase in the above statement is "barring constraints to the contrary". Forensic economists are constrained in their choice of a discount rate. Specifically, *Jones & Laughlin Steel Corp. v. Pfeifer* (103 S. Ct. 2541, or 462 U.S. 523, 1983) found:

> The discount rate should be based on the rate of interest that would be earned on "the best and safest investments." Once it is assumed that the injured worker would definitely have worked for a specific term of years, he is entitled to a risk-free stream of future income to replace his lost wages; therefore, the discount rate should not reflect the market's premium for investors who are willing to accept some risk of default.

Thus, even if the plaintiff can reasonably be expected to invest an award in a portfolio of both risk-free and risky assets, it is clear that the present value of the plaintiff's expected losses should be calculated on the basis of a default-free rate such as a U.S. Treasury rate.[5] Although *Pfeifer* imposes a default-free constraint on the discount rate(s) used in the present value calculations, it is silent on the mix of such qualifying securities. Moreover, because the plaintiff's cash flow needs will almost certainly vary from the projected losses, it is reasonable to expect that the plaintiff will sell securities held in the portfolio before they mature or have excess cash to reinvest. Thus, at a minimum, the forensic economist must be concerned with the expected <u>total</u> return on a portfolio of U.S. Treasuries, or other suitable securities, and the first step in implementing the NDR approach is to identify a proxy, or surrogate, for the total return of such a portfolio.[6]

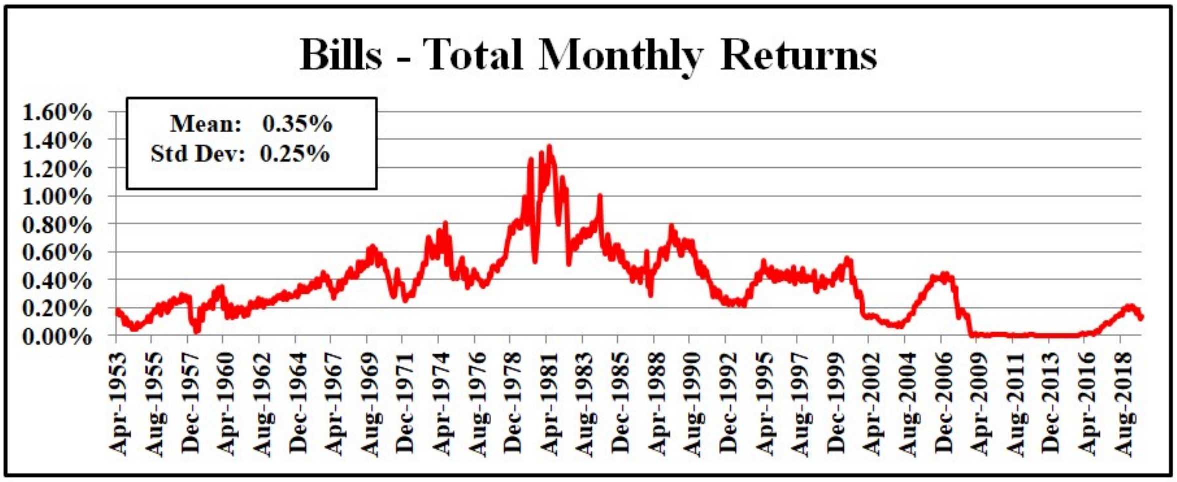
One source of historical data on the total return on a portfolio of U.S. Treasuries is Duff & Phelps' SBBI valuation yearbook: this publication contains the monthly total returns realized from holding portfolios of U.S. Treasury bills, intermediate U.S. bonds (actually, notes), and long-term U.S. Treasury bonds. Specifically, Duff & Phelps' three total-return series correspond to the combined income and capital appreciation returns from holding 1-month Treasury bills, 5-year U.S. Treasury notes, and 20-year U.S. Treasury bonds.[7] The monthly total return for each of these series is shown in Figure 2 below, for the period from April 1953 through December 2019:

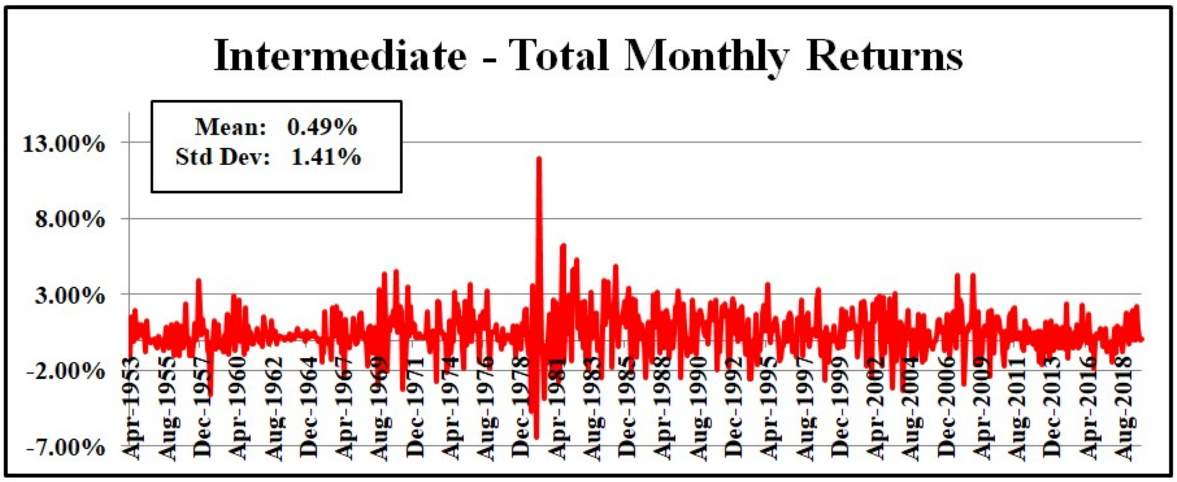
[5] The "best and safest" language in *Pfeifer* is taken from *Chesapeake & Ohio R. Co. v. Kelly* (241 U.S. 485, 1916), which found: ". . . And the putting out of money at interest is at this day so common a matter that ordinarily it cannot be excluded from consideration in determining the present equivalent of future payments, since a reasonable man, even from selfish motives, would probably gain some money by way of interest upon the money recovered. Savings banks and other established financial institutions are in many cases accessible for the deposit of moderate sums at interest, without substantial danger of loss; the sale of annuities is not unknown; and, for larger sums, state and municipal bonds and other securities of almost equal standing are commonly available." Note that the potential financial institution deposits and all of the listed securities have some degree of default risk, as do U.S. Treasury securities. This risk can be diminished through diversification and by selecting only the most credit-worthy issuers.

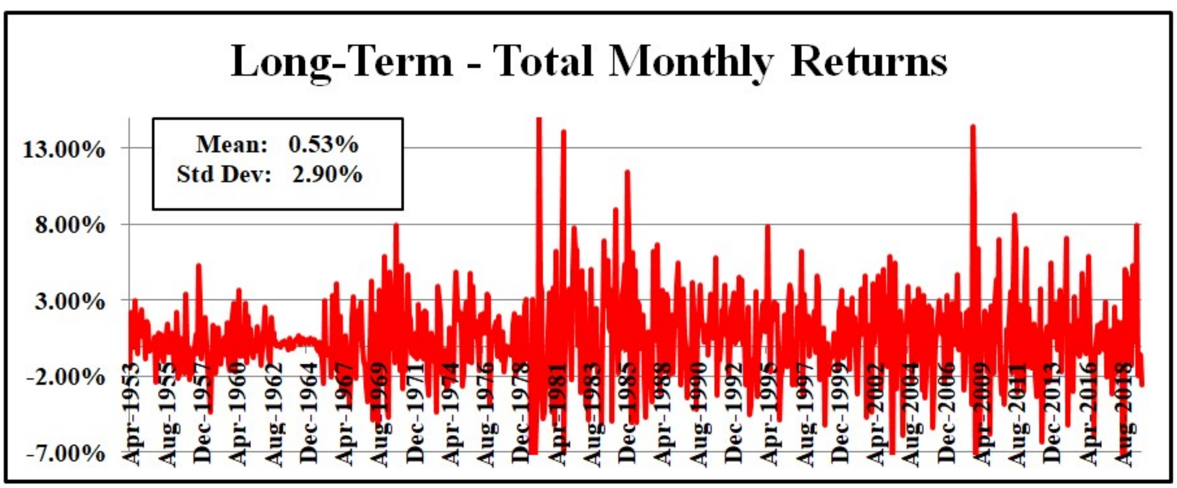
[6] Setting *Pfeifer* aside, another reason to use the return on a portfolio of Treasury securities as the basis for discounting is that it serves as a threshold for evaluating the investments held in the plaintiff's portfolio. That is, it serves a role similar to the cost of capital in a firm's evaluation of a prospective investment project.

[7] The Treasury-bill total return is based on a one-bill portfolio consisting of the bill with the shortest maturity not less than one month. The intermediate-bond total return is based on a one-note portfolio consisting of the noncallable note with the shortest maturity not less than five years. The long-term-bond portfolio is based on a one-bond portfolio consisting of the bond with a maturity of approximately 20 years.

Figure 2 - Duff & Phelps' Bills, Intermediate & Long-Term Returns

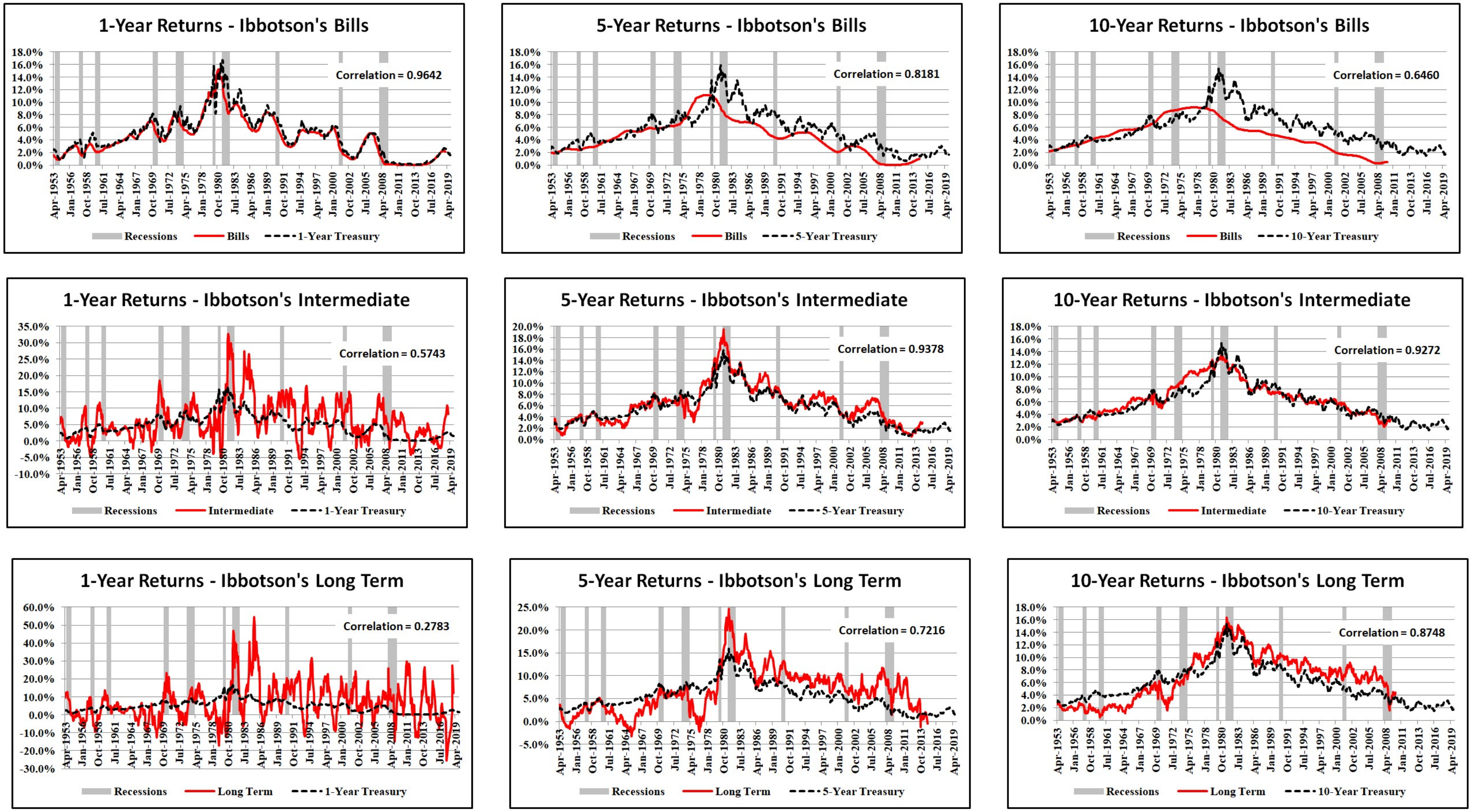






Beyond showing that the intermediate and long-term returns appear to be stationary and more volatile than the total returns for bills, Figure 2 is not very useful. A more interesting portrayal of the data appears in Figure 3 below:

Figure 3 - Rolling Portfolio Returns versus U.S. Treasury Yields



The charts in the first column of Figure 3 compare the 1-year Treasury rate with the subsequent 1-year return of three portfolios consisting entirely of bills, intermediate Treasury securities and long-term Treasury securities, based on Duff & Phelps' monthly total return data. The charts in the center column compare the 5-year Treasury rate with the subsequent 5-year returns of the same portfolios, and the charts the rightmost column make the same comparison for the 10-year Treasury rate and 10-year portfolio returns.

Except for the 1-year return on a portfolio consisting only of Treasury bills, Figure 3 shows that the 1-year Treasury rate is not a good proxy for the expected return on a portfolio of U.S. Treasury securities over a 1-year investment horizon. That is, the 1-year Treasury rate does not track the 1-year return of portfolios consisting of intermediate and long-term securities. However, for the 5- and 10-year horizons, both the 5-year and 10-year Treasury rates track the realized returns whether the portfolio consists entirely of bills, intermediate or long-term securities.

Even if the initial maturity mix of a plaintiff's investment portfolio were known to consist entirely of bills, intermediate Treasury securities or long-term Treasury securities, the portfolio's composition can be expected to change through time as individual holdings age or are sold, and as new securities are purchased. Consequently, because each of the Duff & Phelps series is based on a single maturity, none of them on their own serve as an estimate of the returns that a portfolio of U.S. Treasuries would have produced. Nevertheless, it is possible to gain an understanding of what such a portfolio would have returned by specifying the percentage held in bills and assuming that the remainder of the portfolio was equally split between intermediate and long-term Treasuries as defined by Duff & Phelps. Tables 1 and 2 show the overall fit, as measured by the adjusted R^2, of various estimates of the equation LN(1 + Portfolio Return) = $\alpha + \beta \cdot$LN(1 + Treasury rate). Specifically, the returns of five Treasury bond portfolios were estimated over three investment horizons based on the Duff & Phelps total monthly return data for bills, intermediate and long-term Treasury securities. The five portfolios corresponded to bill percentages of 0, 5, 10, 15 and 20 percent with the remainder of each portfolio evenly split between intermediate and long-term Treasury securities. The portfolio returns were estimated over 1-, 5- and 10-year investment horizons. For each portfolio and investment horizon combination, the above equation was estimated over five sample periods.

For Table 1, the sample periods were: (1) all available data; (2) January 1990 on; and (3) January 2001 on. For the "all available data" sample period, the initial observation corresponds to April 1953. For the 1-year, 5-year and 10-year portfolio returns, the ending observation corresponds to January 2019, January 2015, and to January 2010, respectively. For Table 2, the estimates' sample periods were truncated at the ending observation for the 10-year portfolio returns, January 2010. This truncation produced two additional sample periods, for a total of five.

In both tables it is seen that the ability of Treasury rates to predict future portfolio returns increases markedly when the investment horizon is extended beyond one year. Additionally, there is a slight increase in predictive ability as the percent of the portfolio held in bills increases for all three sample periods. (The exceptions to this result occur in the rightmost column of the two tables.) Finally, with two exceptions, the predictive ability over all sample periods is greatest for the 10-year Treasury rate. The first exception is found in the center of both tables. Here, for the 5-year return and for the subperiod starting in January 1990, the predictive ability is greatest for the 5-year Treasury rate. The second exception is seen in Table 2 for the 5-year return and for the subperiod starting in January 2001. Note that for all estimates the difference in the predictive ability of the 5- and 10-year Treasury rates is not substantial and that both surpass the 1-year Treasury rate by a wide margin. That is, both the 5- and 10-year rates are viable proxies for the return that a portfolio of U.S. Treasuries will produce, whereas the 1-year rate is not.

Table 1 – Adjusted R^2 for Estimates of LN(1 + n-Year Portfolio Return) = $\alpha + \beta \cdot$LN(1 + Treasury rate)

	Dependent is LN(1 + 1-Year Total Return)			Dependent is LN(1 + 5-Year Total Return)			Dependent is LN(1 + 10-Year Total Return)		
	Adjusted R-Squared			Adjusted R-Squared			Adjusted R-Squared		
	All Data	1990-01 On	2001-01 On	All Data	1990-01 On	2001-01 On	All Data	1990-01 On	2001-01 On
				Independent Variable is LN(1 + 1-Year Treasury Rate)					
Bills = 0%	0.1489	0.1461	0.0232	0.6159	0.7324	0.5751	0.6978	0.5086	0.2227
Bills = 5%	0.1640	0.1572	0.0274	0.6415	0.7445	0.5857	0.7119	0.5105	0.2144
Bills = 10%	0.1810	0.1697	0.0323	0.6678	0.7558	0.5961	0.7250	0.5117	0.2056
Bills = 15%	0.2005	0.1841	0.0382	0.6944	0.7661	0.6061	0.7369	0.5124	0.1962
Bills = 20%	0.2229	0.2006	0.0453	0.7211	0.7752	0.6155	0.7471	0.5124	0.1864
				Independent Variable is LN(1 + 5-Year Treasury Rate)					
Bills = 0%	0.2259	0.2338	0.1234	0.6996	0.7801	0.6511	0.8186	0.7587	0.5284
Bills = 5%	0.2431	0.2463	0.1310	0.7252	0.7972	0.6687	0.8297	0.7650	0.5202
Bills = 10%	0.2624	0.2603	0.1397	0.7512	0.8138	0.6867	0.8394	0.7706	0.5112
Bills = 15%	0.2840	0.2760	0.1497	0.7771	0.8296	0.7050	0.8472	0.7752	0.5013
Bills = 20%	0.3084	0.2938	0.1611	0.8028	0.8444	0.7234	0.8528	0.7788	0.4904
				Independent Variable is LN(1 + 10-Year Treasury Rate)					
Bills = 0%	0.2530	0.2786	0.2243	0.7401	0.7592	0.6620	0.8677	0.7970	0.6731
Bills = 5%	0.2704	0.2909	0.2326	0.7646	0.7770	0.6799	0.8759	0.8061	0.6681
Bills = 10%	0.2898	0.3046	0.2418	0.7891	0.7945	0.6983	0.8823	0.8143	0.6623
Bills = 15%	0.3116	0.3198	0.2522	0.8134	0.8114	0.7170	0.8866	0.8217	0.6556
Bills = 20%	0.3360	0.3369	0.2640	0.8371	0.8273	0.7358	0.8883	0.8280	0.6479

Table 2 – Adjusted R^2 for Estimates of LN(1 + n-Year Portfolio Return) = $\alpha + \beta \cdot$LN(1 + Treasury rate)
(All sample periods end in January 2010)

	Dependent is LN(1 + 1-Year Total Return)			Dependent is LN(1 + 5-Year Total Return)			Dependent is LN(1 + 10-Year Total Return)		
	Adjusted R-Squared			Adjusted R-Squared			Adjusted R-Squared		
	1953-04 to 2010-01	1990-01 to 2010-01	2001-01 to 2010-01	1953-04 to 2010-01	1990-01 to 2010-01	2001-01 to 2010-01	1953-04 to 2010-01	1990-01 to 2010-01	2001-01 to 2010-01
	Independent Variable is LN(1 + 1-Year Treasury Rate)								
Bills = 0%	0.1569	0.1207	0.0080	0.6145	0.6554	0.4831	0.6978	0.5086	0.2227
Bills = 5%	0.1713	0.1302	0.0122	0.6380	0.6693	0.4936	0.7119	0.5105	0.2144
Bills = 10%	0.1876	0.1411	0.0174	0.6622	0.6813	0.5031	0.7250	0.5117	0.2056
Bills = 15%	0.2061	0.1537	0.0238	0.6867	0.6907	0.5111	0.7369	0.5124	0.1962
Bills = 20%	0.2273	0.1682	0.0320	0.7114	0.6972	0.5166	0.7471	0.5124	0.1864
	Independent Variable is LN(1 + 5-Year Treasury Rate)								
Bills = 0%	0.2442	0.2208	0.0973	0.7045	0.6746	0.3816	0.8186	0.7587	0.5284
Bills = 5%	0.2605	0.2306	0.1016	0.7282	0.7007	0.4040	0.8297	0.7650	0.5202
Bills = 10%	0.2789	0.2428	0.1115	0.7523	0.7254	0.4234	0.8394	0.7706	0.5112
Bills = 15%	0.2996	0.2566	0.1231	0.7765	0.7486	0.4457	0.8472	0.7752	0.5013
Bills = 20%	0.3228	0.2723	0.1367	0.8006	0.7695	0.4682	0.8528	0.7788	0.4904
	Independent Variable is LN(1 + 10-Year Treasury Rate)								
Bills = 0%	0.2623	0.2459	0.1825	0.7347	0.6191	0.2761	0.8677	0.7970	0.6731
Bills = 5%	0.2786	0.2557	0.1913	0.7579	0.6478	0.2979	0.8759	0.8061	0.6681
Bills = 10%	0.2969	0.2666	0.2014	0.7814	0.6760	0.3218	0.8823	0.8143	0.6623
Bills = 15%	0.3172	0.2789	0.2129	0.8048	0.7032	0.3478	0.8866	0.8217	0.6556
Bills = 20%	0.3400	0.2928	0.2261	0.8278	0.7285	0.3756	0.8883	0.8280	0.6479

Specification of the Net Discount Rate

The proper specification of a net discount rate depends on many factors, not the least of which are the nature of the loss and the characteristics of the plaintiff. For example, if the expected present value of future earnings is the focus of interest, then the growth rate underlying the NDR should correspond to earnings growth. Alternatively, if the focus is on the expected present value of future medical care as specified in a life care plan, the underlying growth rate should depend on the nature of specific life care plan components – growth in the medical CPI for physician services would be appropriate for components dealing with visits to a physician, while growth in the medical CPI for services provided by other medical personnel would be appropriate for components dealing with services provided by a nurse or with physical or occupational therapy.

While the results presented in Tables 1 and 2 suggest that either the 5- or 10-year Treasury rate is a better candidate for the interest rate underlying the chosen NDR than is the 1-year rate, those results do not offer a clear cut choice between these two maturities. This choice may be informed by the characteristics of the plaintiff. For example, in evaluating the present value of a life care plan for a plaintiff with high mortality risk, use of the 5-year Treasury rate may be more appropriate than using the 10-year maturity. With respect to the expected present value of an earnings loss, the plaintiff's remaining work life expectancy may drive the choice between the 5- and 10-year maturities.[8]

In addition to the growth and interest rates underlying the specification of the NDR, a choice between nominal and real growth and interest rates must be made. Note that there is no difference in the NDR based on nominal growth and interest rates and the NDR based on real growth and interest rates if the real interest rate is defined in an *ex post* sense using realized inflation. However, if the real interest is defined in an *ex ante* sense using expected inflation, the two resulting NDRs will be different. To see this, let G and R be the nominal growth and interest rates, and g and r be the real growth and interest rates. Letting ρ and ρ^e be the realized and expected inflation rates we have the following:

$$(1)\quad 1 + NDR_{Nominal} = (1 + R)/(1 + G);$$

$$(2)\quad 1 + NDR_{Real} = (1 + r)/(1 + g);$$

$$(3)\quad 1 + g = (1 + G)/(1 + \rho); \text{ and}$$

$$(4)\quad 1 + r = (1 + R)/(1 + \rho^e);$$

Substituting (3) and (4) into (2) gives

$$(5)\quad 1 + NDR_{Real} = \frac{[(1 + R)/(1 + \rho^e)]}{[(1 + G)/ (1 + \rho)]}$$

$$= [(1 + R)/(1 + G)]\cdot[(1 + \rho)/(1 + \rho^e)]$$

which equals $1 + NDR_{Nominal}$ times one plus the geometric difference between realized and expected inflation. This result is different than that for the nominal NDR based on realized inflation, since if the real interest rate is defined in an *ex post* sense using realized inflation, ρ replaces ρ^e in (5) and it is clear that the two NDR calculations produce the same result.

[8] Of course, nothing prevents a forensic economist from choosing a longer maturity, say 20 or 30 years, if the expected loss period is sufficiently long. Indeed, in the case of an earnings loss, the case could be made that the loss period extends beyond work life expectancy since some portion of the future lost earnings would have presumably funded the plaintiff's retirement.

Figure 4 – Alternative NDRs based on the CPI for Other Medical Personnel

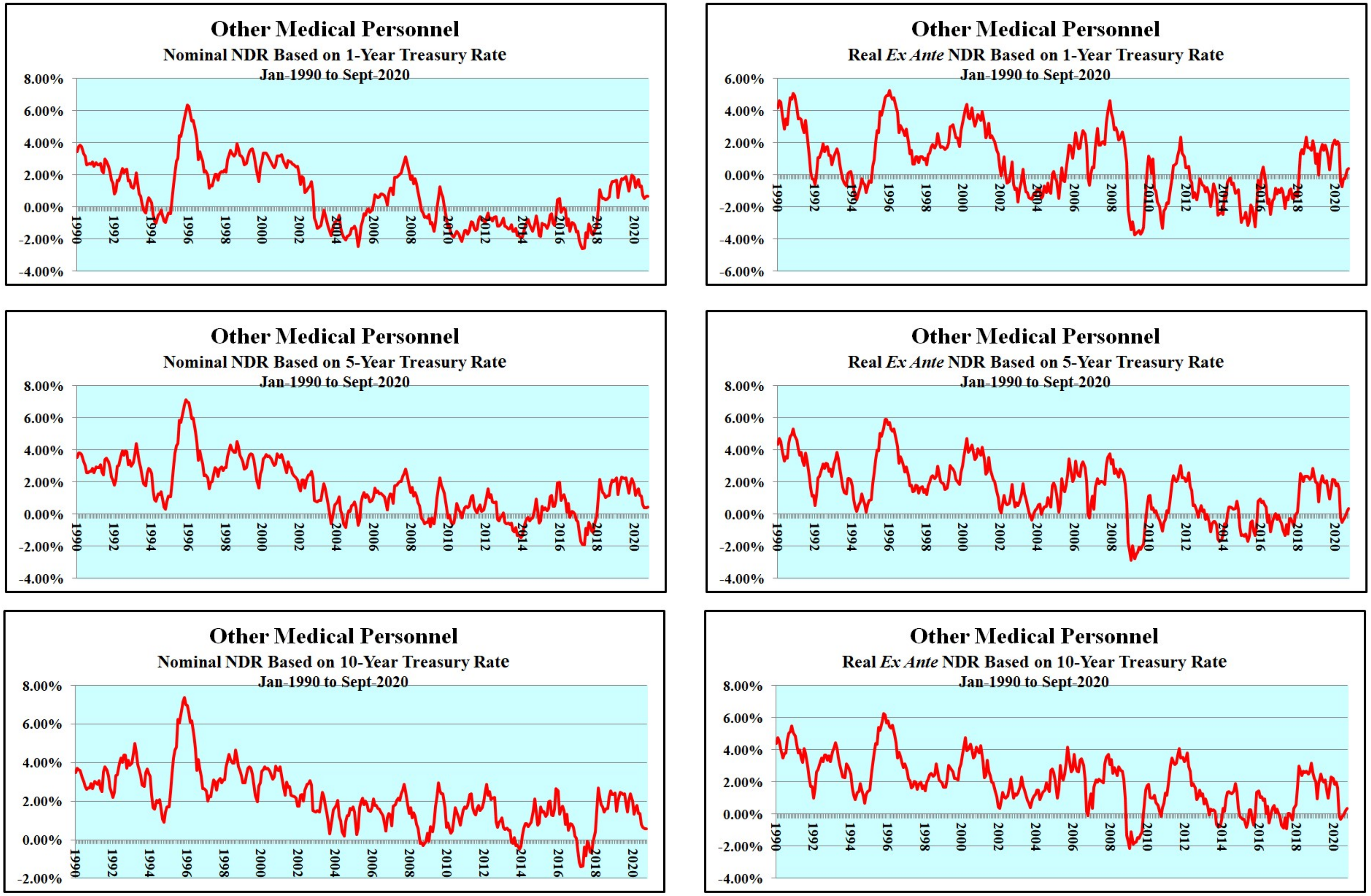


Figure 4 shows six alternative NDRs based on the medical CPI for Other Medical Personnel and on the nominal and real rates for 1-, 5- and 10-year interest rates. The real rates are *ex ante* rates, based on the estimates of expected inflation published by the Federal Reserve Bank of Cleveland. For example, the real 1-year rate is the geometric difference between the nominal 1-year Treasury rate for a given month and the then-current estimate of expected inflation over a 1-year period. The real 5- and 10-year rates are calculated in a similar fashion, using the geometric difference between the 5- and 10-year nominal rates and the then-current estimate of expected inflation for a 5- and 10-year period, respectively. In Figure 4, the nominal NDRs are shown on the left half of the page and the real, *ex ante*, NDRs appear on the right. For each NDR, there appears to be a slight downward trend which is more pronounced for the nominal NDRs. However, the trend is not pronounced enough to conclude that any of the NDRs are nonstationary. Finally, because there is nothing remarkable about the NDRs shown in Figure 4 to favor one over another, the choice must rely on the nature of the loss and characteristics of the plaintiff, and on the stationarity test results.

Testing for Stationarity

There are multiple formal statistical tests available to a forensic economist to assist in deciding whether a particular NDR is stationary. These tests differ both in the assumption made concerning the underlying process and in the specification of the null hypothesis. For example, the augmented Dickey-Fuller (ADF) test tests the null hypothesis that $\rho=1$ in the equation

$Y_t = \alpha + \rho \cdot Y_{t-1} + \sum_{j=1}^{k} \lambda_j \Delta Y_{t-j} + \varepsilon_t$ where Y is the NDR, Δ is the first difference operator, and ε_t is a random error term. By comparison, the Phillips-Perron (PP) test tests the same null hypothesis for the equation

$Y_t = \alpha + \rho \cdot Y_{t-1} + \varepsilon_t$ but corrects for autocorrelation and heteroscedasticity in the error term. For both the ADF and PP tests, rejecting the null that $\rho=1$ supports the conclusion that Y is stationary.

The Kwiatkowski, Phillips, Schmidt and Shinn (KPSS) test tests the null that the series of interest is stationary. Consequently, being unable to reject the null provides support for the conclusion that the NDR in question is stationary. The KPSS test results provide four reference points of how low the confidence level must be in order to be able to reject the null:

(1) greater than 99 percent;

(2) between 95 and 99 percent;

(3) between 90 and 95 percent; and

(4) less than 90 percent.

The strongest support for the stationary conclusion is given by the last of these four reference points.

For a given set of observations, it may well be that different stationarity tests will lead to a different conclusion with regard to the stationarity of the NDR. Consequently, the forensic economist should not rely on the results of a single test but instead be informed by the results from several tests. With respect to the NDRs in Figure 4, the stationarity issue was examined using four formal stationarity tests: the ADF test, two PP tests and the KPSS test.[9] In addition, the correlogram of each NDR was examined, along with the ordinary-least-squares estimate of ρ and it standard error in the equation

$Y_t = \alpha + \rho \cdot Y_{t-1} + \varepsilon_t$ with both the estimate of ρ and its standard error corrected for bias.

[9] All four tests were performed using version 11 of E-Views. The difference between the two PP tests lies in the technique to estimate the distribution of the error term in the PP test equation. The first PP test utilized E-Views' Bartlett kernel default, while the second utilized an autoregressive specification for the error term.

With respect to the correlogram, if the sample autocorrelations decay to zero, then the time series in question can be viewed as stationary. (See Box and Jenkins (1971), Nielsen (2006) and Enders (2010) for examples of autocorrelation functions for stationary and non-stationary series.) Likewise, the difference between one and the absolute value of the corrected estimate of ρ, measured in terms of its corrected standard error, informs the issue of the NDR's stationarity – the greater the distance, the greater the support for the conclusion that the NDR is stationary.[10]

The results of these tests for the nominal NDRs are summarized in Table 3 below:

Table 3 - Summary of Stationarity Test Results - Nominal NDRs

Nominal NDR Based on 1-Year Treasury Rate

Sample Period	Correlogram Dies Out After	Conf Level for ADF Test	Conf Level for PP Test (Bartlett Kernel)	Conf Level for PP Test (AR OLS)	Conf Level for KPSS H_0: NDR is stationary	$(1 - \lvert\hat{\rho}\rvert) \div SE\hat{\rho}$ Based on Corrected OLS Est
1990M01 2020M09	17	87.4%	91.2%	93.0%	> 99%	1.24
1990M01 2000M12	7	78.7%	78.1%	77.5%	< 90%	0.30
2001M01 2020M09	11	97.5%	95.2%	97.5%	< 90%	1.63

Nominal NDR Based on 5-Year Treasury Rate

Sample Period	Correlogram Dies Out After	Conf Level for ADF Test	Conf Level for PP Test (Bartlett Kernel)	Conf Level for PP Test (AR OLS)	Conf Level for KPSS H_0: NDR is stationary	$(1 - \lvert\hat{\rho}\rvert) \div SE\hat{\rho}$ Based on Corrected OLS Est
1990M01 2020M09	19 (plus 23-36)	98.3%	96.3%	98.2%	> 99%	1.78
1990M01 2000M12	5 (plus 14-17)	93.6%	91.1%	93.9%	< 90%	0.90
2001M01 2020M09	9	99.9%	99.6%	99.9%	< 95% & >90%	2.77

Nominal NDR Based on 10-Year Treasury Rate

Sample Period	Correlogram Dies Out After	Conf Level for ADF Test	Conf Level for PP Test (Bartlett Kernel)	Conf Level for PP Test (AR OLS)	Conf Level for KPSS H_0: NDR is stationary	$(1 - \lvert\hat{\rho}\rvert) \div SE\hat{\rho}$ Based on Corrected OLS Est
1990M01 2020M09	12 (plus 24-36)	99.1%	98.1%	99.1%	> 99%	2.13
1990M01 2000M12	5 (plus 13-18)	92.4%	92.1%	93.1%	< 90%	0.94
2001M01 2020M09	7	100.0%	99.9%	100.0%	< 95% & >90%	3.40

[10]Orcutt and Winokur (1969) provide estimates of ρ and its variance that correct for the bias in the OLS estimates. In the results presented below, the estimated values of ρ were positive and less than one in all cases. While an absolute value less than one is a requirement for a stationary series, in and of itself, it does not add support to the conclusion that the NDR is stationary. However, a large distance between one and the absolute value of the corrected estimate of ρ, measured in terms of its corrected standard error, does.

The results of these tests for the real, *ex ante* NDRs are summarized in Table 4 below:

Table 4 - Summary of Stationarity Test Results - Real *Ex Ante* NDRs

Real *Ex Ante* NDR Based on 1-Year Treasury Rate

Sample Period	Correlogram Dies Out After	Conf Level for ADF Test	Conf Level for PP Test (Bartlett Kernel)	Conf Level for PP Test (AR OLS)	Conf Level for KPSS H_0: NDR is stationary	$(1 - \lvert\hat{\rho}\rvert) \div SE\hat{\rho}$ Based on Corrected OLS Est
1990M01 2020M09	11	99.7%	98.5%	99.6%	> 99%	2.42
1990M01 2000M12	7	64.9%	79.5%	64.9%	< 90%	0.57
2001M01 2020M09	8	99.7%	99.4%	100.0%	< 90%	2.50

Real *Ex Ante* NDR Based on 5-Year Treasury Rate

Sample Period	Correlogram Dies Out After	Conf Level for ADF Test	Conf Level for PP Test (Bartlett Kernel)	Conf Level for PP Test (AR OLS)	Conf Level for KPSS H_0: NDR is stationary	$(1 - \lvert\hat{\rho}\rvert) \div SE\hat{\rho}$ Based on Corrected OLS Est
1990M01 2020M09	11	97.9%	99.2%	98.8%	> 99%	2.41
1990M01 2000M12	6	92.8%	88.5%	91.2%	< 90%	0.91
2001M01 2020M09	7	99.8%	99.6%	99.8%	< 90%	2.60

Real *Ex Ante* NDR Based on 10-Year Treasury Rate

Sample Period	Correlogram Dies Out After	Conf Level for ADF Test	Conf Level for PP Test (Bartlett Kernel)	Conf Level for PP Test (AR OLS)	Conf Level for KPSS H_0: NDR is stationary	$(1 - \lvert\hat{\rho}\rvert) \div SE\hat{\rho}$ Based on Corrected OLS Est
1990M01 2020M09	10	98.7%	99.6%	99.5%	> 99%	2.13
1990M01 2000M12	6	92.5%	89.8%	91.3%	< 90%	0.96
2001M01 2020M09	6	99.9%	99.7%	99.9%	< 90%	2.80

In both tables, the first column shows the number of months it takes for the correlation between the NDR and its lagged values to die out. An entry like "19 (plus 23-36)" indicates that the lagged correlations die out after 19 months and then increase to a significant level in months 23 through 36.[11] The next three columns show the confidence level at which the null hypothesis that ρ=1 (*i.e.*, that the NDR is not stationary) is rejected for the ADF and two PP tests. The higher the confidence level, the greater is the support for the conclusion that the NDR is stationary. The next column shows how low the confidence level must be in order to reject the KPSS test's null hypothesis that the NDR is stationary. Here, the lower the confidence interval, the greater is the support for the conclusion that the NDR is stationary. Finally, the last column shows how far the corrected coefficient of the NDR lagged one month is from 1, in terms of its corrected standard error. Again, the greater this distance is, the greater is the support for the conclusion that the NDR is stationary.

Tables 5 and 6 summarize the above results in terms of whether or not they support the conclusion that the NDR measure is stationary. In the first column, the support is characterized as "Strong" if the lagged correlation dies out within a year. If the lagged correlation dies out over a longer period, the support is characterized as "Moderate". No entry appears if the lagged correlation dies out and subsequently returns to a significant level. For the ADF and PP tests, the support is characterized as "Strong" if the confidence level is greater than or equal to 95 percent. It is characterized as "Moderate" if the confidence level is between 90 and 95 percent. The support is characterized as "Weak" for confidence levels that are

[11] A lagged correlation is deemed to be significant if it falls outside of a 95 percent confidence level centered on zero.

between 80 and 90 percent. No entry appears for all other values. For the KPSS test, the support is characterized as “Strong” if the confidence level required to reject the null hypothesis is below 90 percent. It is characterized as “Moderate” if the confidence level required to reject the null hypothesis is between 90 and 95 percent. No entry appears for all other outcomes. Finally, for the last column, the support is characterized as “Strong” if the corrected estimate of ρ is more than 2.5 corrected standard errors from 1. The support is characterized as “Moderate” if the distance from 1 is between 2.0 and 2.5 corrected standard errors. If the distance is between 1.5 and 2.0 corrected standard errors, the support is characterized as “Weak”. No entry appears for all other outcomes.

Table 5 – Support for Stationarity Conclusion - Nominal NDRs

Nominal NDR Based on 1-Year Treasury Rate

Sample Period	Correlogram Dies Out After	Conf Level for ADF Test	Conf Level for PP Test (Bartlett Kernel)	Conf Level for PP Test (AR OLS)	Conf Level for KPSS H_0: NDR is stationary	$(1 - \lvert\hat{\rho}\rvert) \div SE\hat{\rho}$ Based on Corrected OLS Est
1990M01 2020M09	Moderate	Weak	Moderate	Moderate		
1990M01 2000M12	Strong				Strong	
2001M01 2020M09	Strong	Strong	Strong	Strong	Strong	Weak

Nominal NDR Based on 5-Year Treasury Rate

Sample Period	Correlogram Dies Out After	Conf Level for ADF Test	Conf Level for PP Test (Bartlett Kernel)	Conf Level for PP Test (AR OLS)	Conf Level for KPSS H_0: NDR is stationary	$(1 - \lvert\hat{\rho}\rvert) \div SE\hat{\rho}$ Based on Corrected OLS Est
1990M01 2020M09		Strong	Strong	Strong		Weak
1990M01 2000M12		Moderate	Moderate	Moderate	Strong	
2001M01 2020M09	Strong	Strong	Strong	Strong	Moderate	Strong

Nominal NDR Based on 10-Year Treasury Rate

Sample Period	Correlogram Dies Out After	Conf Level for ADF Test	Conf Level for PP Test (Bartlett Kernel)	Conf Level for PP Test (AR OLS)	Conf Level for KPSS H_0: NDR is stationary	$(1 - \lvert\hat{\rho}\rvert) \div SE\hat{\rho}$ Based on Corrected OLS Est
1990M01 2020M09		Strong	Strong	Strong		Moderate
1990M01 2000M12		Moderate	Moderate	Moderate	Strong	
2001M01 2020M09	Strong	Strong	Strong	Strong	Moderate	Strong

Table 6 – Support for Stationarity Conclusion - Real *Ex Ante* NDRs

Real *Ex Ante* NDR Based on 1-Year Treasury Rate

Sample Period	Correlogram Dies Out After	Conf Level for ADF Test	Conf Level for PP Test (Bartlett Kernel)	Conf Level for PP Test (AR OLS)	Conf Level for KPSS H_0: NDR is stationary	$(1 - \lvert\hat{p}\rvert) \div SE\hat{p}$ Based on Corrected OLS Est
1990M01 2020M09	Strong	Strong	Strong	Strong		Moderate
1990M01 2000M12	Strong				Strong	
2001M01 2020M09	Strong	Strong	Strong	Strong	Strong	Moderate

Real *Ex Ante* NDR Based on 5-Year Treasury Rate

Sample Period	Correlogram Dies Out After	Conf Level for ADF Test	Conf Level for PP Test (Bartlett Kernel)	Conf Level for PP Test (AR OLS)	Conf Level for KPSS H_0: NDR is stationary	$(1 - \lvert\hat{p}\rvert) \div SE\hat{p}$ Based on Corrected OLS Est
1990M01 2020M09	Strong	Strong	Strong	Strong		Moderate
1990M01 2000M12	Strong	Moderate	Weak	Moderate	Strong	
2001M01 2020M09	Strong	Strong	Strong	Strong	Strong	Strong

Real *Ex Ante* NDR Based on 10-Year Treasury Rate

Sample Period	Correlogram Dies Out After	Conf Level for ADF Test	Conf Level for PP Test (Bartlett Kernel)	Conf Level for PP Test (AR OLS)	Conf Level for KPSS H_0: NDR is stationary	$(1 - \lvert\hat{p}\rvert) \div SE\hat{p}$ Based on Corrected OLS Est
1990M01 2020M09	Strong	Strong	Strong	Strong		Moderate
1990M01 2000M12	Strong	Moderate	Weak	Moderate	Strong	
2001M01 2020M09	Strong	Strong	Strong	Strong	Strong	Strong

Tables 5 and 6 show that support for the conclusion that the NDR is stationary is strongest for the January 2001 to September 2020 period, for both the nominal and real, *ex ante*, NDRs. While the support is slightly weaker for the 1-year nominal NDR than for the corresponding real, *ex ante*, NDR, the difference is not that striking and, other than choosing results based on the more recent data, these tables give no clear guidance on which NDR should be preferred. Ultimately, the decision may be driven by how much difference the choice between the NDRs makes. Tables 7 and 8 address this question:

Table 7 - Estimated Values of the Long-Run Net Discount Rate

	Long-Run NDRs Base On								
	1-Year Treasury Rate				5-Year Treasury Rate		10-Year Treasury Rate		
Autoregressive Model	**Nominal**		**Real *Ex Ante***		**Nominal**	**Real *Ex Ante***	**Nominal**	**Real *Ex Ante***	
					1990M01 2020M09				
Geometric Mean	0.718%		0.669%		1.602%	1.511%	2.126%	1.984%	
AR(1)	0.968%		0.841%		1.662%	1.601%	2.125%	2.029%	
AR(1), AR(2)	0.893%		0.799%		1.645%	1.574%	2.127%	2.017%	
AR(1), AR(2), AR(3)	0.892%		0.795%		1.646%	1.577%	2.127%	2.019%	
					1990M01 2000M12				
Geometric Mean	2.329%		2.040%		3.184%	2.829%	3.485%	3.159%	
AR(1)	2.596%		2.485%		3.254%	3.067%	3.504%	3.340%	
AR(1), AR(2)	2.494%		2.355%		3.220%	2.965%	3.494%	3.266%	
AR(1), AR(2), AR(3)	2.447%		2.284%		3.207%	2.930%	3.490%	3.242%	
					2001M01 2020M09				
Geometric Mean	-0.158%		-0.078%		0.742%	0.792%	1.386%	1.344%	
AR(1)	0.224%	*	0.139%	*	0.861%	0.925%	1.432%	1.420%	
AR(1), AR(2)	0.124%	*	0.083%	*	0.826%	0.886%	1.419%	1.402%	
AR(1), AR(2), AR(3)	0.167%	*	0.082%	*	0.838%	0.893%	1.423%	1.404%	

*** Autoregressive estimate not significantly different from zero.**

Table 7 shows various estimates of the nominal and real, *ex ante*, NDRs based on the geometric mean and on three autoregressive model specifications.[12] Several patterns emerge from Table 7. First, regardless of how the NDR is estimated both the nominal and real, *ex ante*, NDRs increase as the maturity of the underlying Treasury security increases. Second, for the NDR based on the 10-year Treasury rate, the real, *ex ante*, NDR is less than the corresponding nominal NDR for all three periods. The same is true for the NDR based on the 5-year Treasury rate, except for the January 2001 to September 2020 subperiod. For the NDR based on the 1-year Treasury rate, the real, *ex ante*, NDR is less than the nominal NDR in all cases except for the geometric mean estimate for the January 2001 to September 2020 subperiod. Here, the geometric mean estimates of the NDRs are negative and the autoregressive estimates are not significantly different from zero. Finally, the nominal/real differential is smallest for the NDR based on the 10-year Treasury rate in the most recent subperiod and is likely not material.

Table 8 shows the autoregressive estimates of the long-run NDRs minus the corresponding geometric mean. This difference decreases as the maturity of the underlying Treasury security increases for all three autoregressive specifications. Also, for the real, *ex ante*, NDRs, the differential is smallest for the NDR based on the 10-year Treasury rate for the entire period, although it is less than 10 basis points in the most recent subperiod. For the nominal NDRs, the differential is largest in the most recent subperiod, but it is less than 5 basis points for all three autoregressive specifications.

[12]Details underlying the autoregressive estimates are presented in a separate appendix at the end of this paper. For the nominal NDRs, the adjusted R^2s range between 77 and 86 percent. For the real, *ex ante*, NDRs, the range is between 80 and 92 percent.

Table 8 - Autoregressive Estimates of the Long-Run NDRs minus Geometric Mean (Basis Points)

	Long-Run NDRs Base On							
	1-Year Treasury Rate				5-Year Treasury Rate		10-Year Treasury Rate	
Autoregressive Model	Nominal		Real *Ex Ante*		Nominal	Real *Ex Ante*	Nominal	Real *Ex Ante*
					1990M01 2020M09			
AR(1)	25.0		17.2		5.9	9.1	-0.1	4.5
AR(1), AR(2)	17.5		13.0		4.3	6.4	0.1	3.3
AR(1), AR(2), AR(3)	17.4		12.6		4.3	6.6	0.0	3.4
					1990M01 2000M12			
AR(1)	26.7		44.4		7.1	23.8	1.8	18.1
AR(1), AR(2)	16.4		31.4		3.6	-10.1	0.9	10.7
AR(1), AR(2), AR(3)	11.8		24.4		2.3	-3.6	0.4	8.3
					2001M01 2020M09			
AR(1)	38.2	*	21.7	*	11.8	13.3	4.6	7.7
AR(1), AR(2)	28.1	*	16.1	*	8.4	9.4	3.3	5.8
AR(1), AR(2), AR(3)	32.5	*	15.9	*	9.6	10.1	3.6	6.1

* **Autoregressive estimate not significantly different from zero.**

Summary and Conclusions

The main conclusion of this paper is that use of an NDR eliminates the bias in the estimated present value of the plaintiff's future losses. Because no one knows what would have happened but for the issue giving rise to the tort, and because the plaintiff's future investment returns are unknown, the best any forensic economist can do is estimate the expected present value of the plaintiff's future losses. Moreover, even if the future losses and future investment returns equal their respective expected values, the estimated present value of the future losses will be biased when it is calculated using separate growth and discount rates due to the covariance between the investment returns and the growth rate underlying the estimates of the future losses. Provided the NDR is stationary, use of an NDR eliminates this bias because the covariance between the resulting present value factor and the underlying loss estimates is zero.

Implementing the NDR approach involves three separate tasks: (1) identification of the expected returns the plaintiff will receive in the future; (2) determination of an NDR based on this expected return; and (3) testing the NDR for stationarity. Unlike the conclusion that use of an NDR eliminates the bias in the estimate of the expected present value, how to go about completing these tasks is not clear cut.

Pfeifer imposes the requirement that the present value must be based on a discount rate free of default risk and it follows that the interest rate component of the NDR must reflect the return on a portfolio of U.S. Treasury securities. Alternatively, the return of on such a portfolio can be viewed as a threshold for evaluating the investments held in a plaintiff's portfolio in the same way that a firm's cost of capital is used to evaluate a prospective investment project. Based on the Dunn and Phelps total return data and a wide range of portfolio mixes, the analysis presented above suggests that both the 5-year and the 10-year Treasury rates are viable proxies for the return that a portfolio of U.S. Treasuries will produce, whereas the 1-year Treasury rate is not. In addition to a potential proxy's predictive ability, the choice of the maturity of the proxy for the Treasury portfolio return will be driven by the nature of the loss and the characteristics of the plaintiff. Losses expected to occur over a long period may warrant a longer maturity, while those occurring over a shorter period – say, due to the plaintiff's high mortality risk – may warrant a shorter maturity.

When specifying the NDR to be used, the forensic economist must not only pick a growth rate but he/she must also choose between a nominal and a real, *ex ante*, perspective for the interest rate component. Similarly, testing the NDR for stationarity is not a simple, one-step, process since a variety of stationary tests are available which may give conflicting results for the same set of observations. In addition, the both the stationarity test results and the resulting NDR may vary depending on the sample period used in analyzing the NDR.

This paper presented an example of determining an NDR based on the medical CPI for Other Medical Personnel and on the nominal and real, *ex ante*, rates for 1-, 5- and 10-year interest rates, calculated over the January 1990 to September 2020 sample period. The nominal NDR equaled the geometric difference between each respective Treasury rate in a given month and the subsequent 1-year change in the medical CPI. The real, *ex ante*, NDR equaled the geometric difference between each respective *ex ante* Treasury rate – defined as the difference between the nominal rate and then-current expected average level of 1-, 5- and 10-year inflation – and the subsequent 1-year real, or inflation-adjusted, change in the medical CPI. This produced six potential NDRs in all (3 maturities each for the nominal and real, *ex ante*, interest rate components.) The stationarity of each NDR was examined based on the results of four formal stationarity tests, the NDR's correlogram and the distance between one and the corrected estimate of the slope coefficient in a simple autoregressive model. The analysis was conducted over the entire sample period and for two subperiods. Overall support for the conclusion that the NDRs are stationary was weakest for those based on the 1-year Treasury rate, although support for this conclusion was strong across the board for the most recent subperiod (January 2001 to September 2020). However, for this subperiod, the resulting 1-year NDRs were negative when based on the geometric mean and positive but not significantly different from zero when estimated by three autoregressive specifications. Overall, the long-run real, *ex ante*, NDRs are less than the corresponding nominal NDRs, with the differential being smallest (and insubstantial) for the NDRs based on the 10-year Treasury rate for the most recent subperiod.

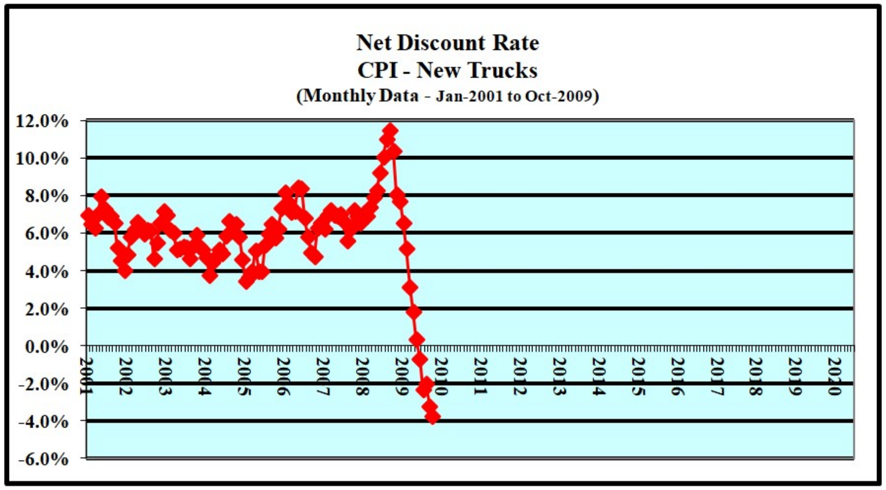
This example demonstrates there is no one-size fits all approach when deciding what NDR to use in a given case. The example also demonstrates that the resulting NDR estimates will vary, depending on the sample period over which they are calculated. The example, and the methodology it illustrates, is silent on what sample period is appropriate – the forensic economist must look to other factors in making this decision.[13]

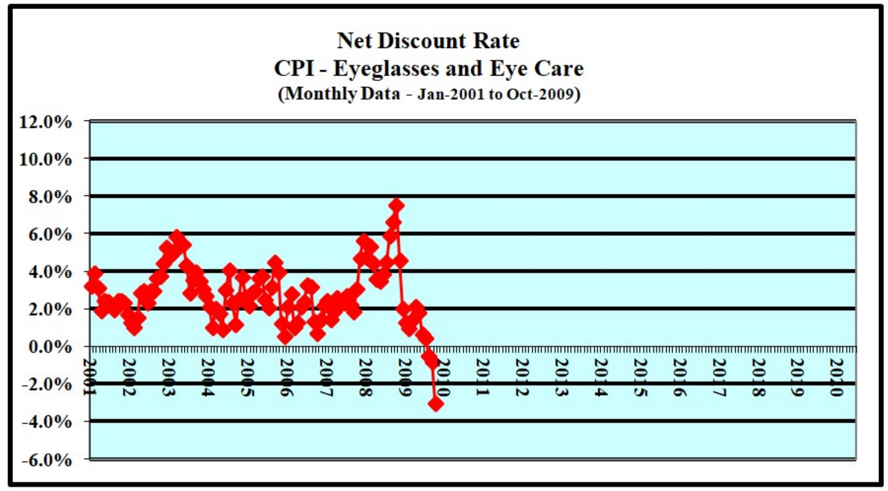
Finally, related to the sample-period issue is the question of whether the selected NDR will remain stationary in the future. The current pandemic and the resulting recession make this question all the more important and, unfortunately, unanswerable. While the data will ultimately speak to this issue, they can only do so with a lag, after a significant amount of time has passed.[14] This is illustrated in Figure 5 below:

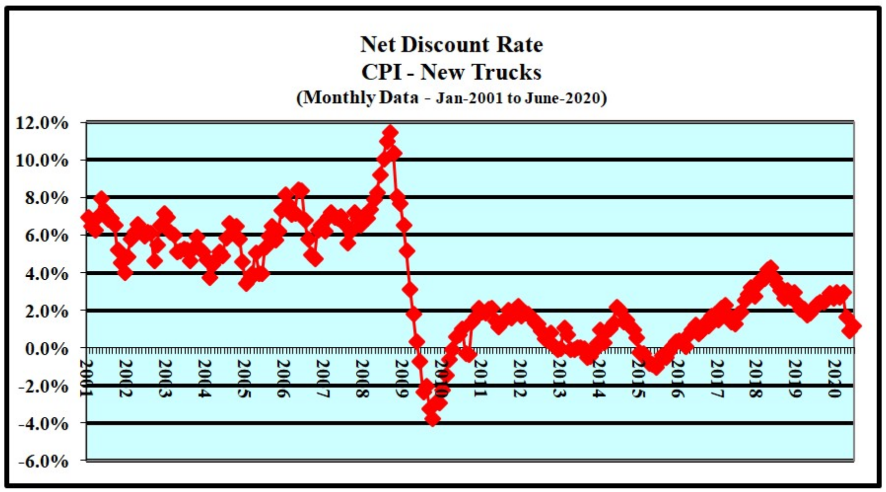
[13]For example, reasons for starting the analysis with 2001 include: (1) a downward shift in the economy's long-run expected growth starting in the late 1990's and early 2000's due to slowing population and productivity growth and to a decline in labor force participation; (2) a sharp downward shift in employment during business expansions since the end of the 2001 recession; and (3) a shift in monetary policy from a rule-based paradigm to a discretionary policymaking regime. With respect to (3), see Taylor (2011).

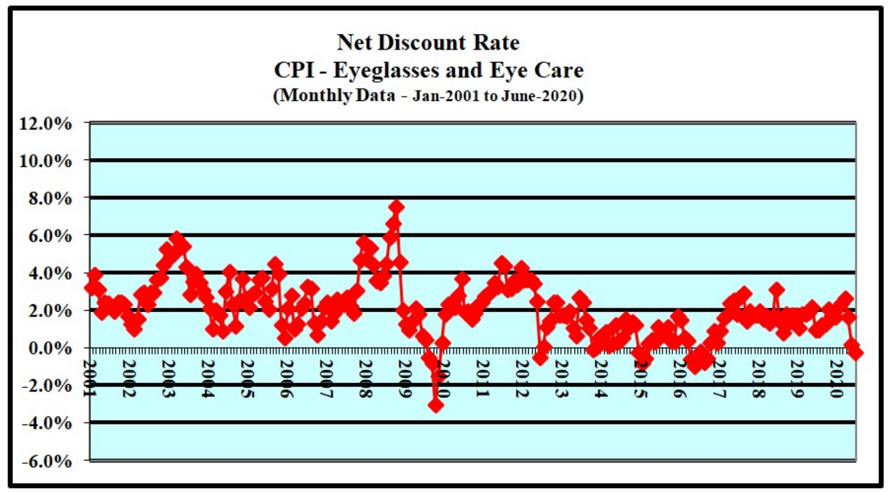
[14]The author credits this observation – the Nieswiadomy Insight – to Dr. Michael Nieswiadomy who made this comment more than a decade ago at theWEAI's 85th Annual Conference in Portland, Oregon.

Figure 5 – The Nieswiadomy Insight









The left column of Figure 5 plots a real, *ex ante* NDR based on the CPI for new trucks; the right column plots a real, *ex ante*, NDR based on the medical CPI for eye glasses and eye care. As shown in the upper half of Figure 5, both NDRs experienced a sharp decline ending in October 2009, shortly after the end of the 2007-2009 Great Recession. At that point in time, it was not be possible to conclude on the basis of the data alone whether or not the decline represented a permanent break in the series or if the NDR in question would recover and remain stationary. Subsequent experience shows that the former was the case for the new vehicles NDR while the eye care NDR subsequently recovered. Clearly, the issue of whether or not forensic economists can rely on data prior to the pandemic or whether we face uncharted territory cannot be resolved in real time.

Bibliography

Box, George P. and Gwilym M. Jenkins, *Time Series Analysis – Forecasting and Control*, August 1971, Holden Day, Cambridge.

Chesapeake & Ohio R. Co. v. Kelly (241 U.S. 485, 1916)

Duff & Phelps. 2020. *2020 SBBI Yearbook: Stocks, Bonds, Bills and Inflation*.

Enders, Walter, *Applied Econometric Time Series*, 3rd Edition, 2010, John Wiley and Sons, Inc., Hoboken, NJ.

Jones & Laughlin Steel Corp. v. Pfeifer, 103 S.Ct. 2541 (1983)

Krueger, Kurt V. 2016. "Retiring the Historical Net Discount Rate." *Journal of Forensic Economics*, 26(2):147-180.

Nielsen, Bent, "Corrleograms for Non-Stationary Autoregressions", *Journal of the Royal Statistical Society, Series B*, 68(4), 2006, pp. 707-720.

Nowak, Laura S. 1991. "Empirical Evidence on the Relationship Between Earnings Growth and Interest Rate." *Journal of Forensic Economics*, 4(2):187-202.

Orcutt, Guy H. and Herbert S. Winokur, Jr., "First Order Autoregression Inference, Estimation, and Prediction", *Econometrica*, 37(1), January, 1969, pp. 1-14.

Pelaez, Rolando F. 1991. "Valuation of Earnings Using Historical Growth-Discount Rates." *Journal of Forensic Economics*, 5(1):27-44.

Schap, David, Robert Baumann, and Lauren Guest. 2014. "Wage Net Discount Rates: 1981-2012." *Journal of Forensic Economics*, 25(2):153-174.

Schap, David, and Robert Baumann. 2015. "Medical Net Discount Rates: Updated and Re-examined." *Journal of Forensic Economics*, 26(1):1-16.

Taylor, John B. 2011, "The Cycle of Rules and Discretion in Economic Policy", *National Affairs* (7), pp. 55-65.

Appendix - Details Underlying the Autoregressive Model Estimates of the Long-Run NDRs

NDRs Based on the 1-Year Treasury Rate

	Long Run NDR	AR(1)	AR(2)	AR(3)	Adj R^2	DW	Long Run NDR	AR(1)	AR(2)	AR(3)	Adj R^2	DW
	Nominal NDR Based on 1-Year Treasury Rate						*Real Ex Ante NDR Based on 1-Year Treasury Rate*					
1990M01 2020M09											**# of Observations = 369**	
Estimate	0.968%	0.97521	--	--	0.95065	1.612	0.841%	0.95123	--	--	0.90205	1.712
t-Statistic	1.287	82.123	--	--			1.391	56.273	--	--		
Confidence Level	80.110%	100.000%	--	--			83.486%	100.000%	--	--		
Estimate	0.893%	1.16662	-0.19643	--	0.95243	1.995	0.799%	1.09302	-0.14964	--	0.90397	2.001
t-Statistic	1.392	24.199	-4.240	--			1.502	21.879	-3.117	--		
Confidence Level	83.531%	100.000%	99.997%	--			86.616%	100.000%	99.803%	--		
Estimate	0.892%	1.16616	-0.19370	-0.00235	0.95230	1.994	0.795%	1.09009	-0.12822	-0.01967	0.90375	1.996
t-Statistic	1.391	23.603	-2.688	-0.051			1.520	21.509	-1.688	-0.364		
Confidence Level	83.486%	100.000%	99.248%	4.076%			87.073%	100.000%	90.771%	28.370%		
1990M01 2000M12											**# of Observations = 132**	
Estimate	2.596%	0.95771	--	--	0.92337	1.437	2.485%	0.95498	--	--	0.91049	1.630
t-Statistic	3.051	39.610	--	--			2.839	33.562	--	--		
Confidence Level	99.723%	100.000%	--	--			99.475%	100.000%	--	--		
Estimate	2.494%	1.23166	-0.28423	--	0.92914	2.074	2.355%	1.13161	-0.18553	--	0.91290	2.024
t-Statistic	3.686	15.689	-3.736	--			3.189	13.412	-2.237	--		
Confidence Level	99.966%	100.000%	99.972%	--			99.820%	100.000%	97.301%	--		
Estimate	2.447%	1.18440	-0.08142	-0.16350	0.93056	1.990	2.284%	1.10950	-0.05065	-0.11967	0.91348	2.008
t-Statistic	4.140	13.535	-0.576	-1.988			3.454	13.246	-0.423	-1.450		
Confidence Level	99.994%	100.000%	43.443%	95.108%			99.925%	100.000%	32.670%	85.046%		
2001M01 2020M09											**# of Observations = 237**	
Estimate	0.224%	0.96338	--	--	0.91272	1.714	0.139%	0.92685	--	--	0.84672	1.726
t-Statistic	0.399	57.247	--	--			0.265	37.907	--	--		
Confidence Level	31.005%	100.000%	--	--			20.905%	100.000%	--	--		
Estimate	0.124%	1.11549	-0.15979	--	0.91455	1.995	0.083%	1.07163	-0.15778	--	0.84986	2.017
t-Statistic	0.255	18.585	-2.768	--			0.182	16.765	-2.538	--		
Confidence Level	20.117%	100.000%	99.391%	--			14.389%	100.000%	98.818%	--		
Estimate	0.167%	1.12756	-0.24397	0.07626	0.91468	2.030	0.082%	1.07060	-0.15062	-0.00675	0.84922	2.016
t-Statistic	0.323	18.882	-2.909	1.283			0.178	16.507	-1.534	-0.094		
Confidence Level	25.281%	100.000%	99.602%	79.937%			14.117%	100.000%	87.365%	7.502%		

Appendix - Details Underlying the Autoregressive Model Estimates of the Long-Run NDRs (continued)

NDRs Based on the 5-Year Treasury Rate

Nomimal NDR Based on 5-Year Treasury Rate

	Long Run NDR	AR(1)	AR(2)	AR(3)	Adj R^2	DW
1990M01 2020M09						
Estimate	1.662%	0.96281	--	--	0.92687	1.620
t-Statistic	2.886	68.099	--	--		
Confidence Level	99.586%	100.000%	--	--		
Estimate	1.645%	1.15036	-0.19487	--	0.92947	1.990
t-Statistic	3.393	22.657	-3.977	--		
Confidence Level	99.923%	100.000%	99.992%	--		
Estimate	1.646%	1.15239	-0.20682	0.01038	0.92928	1.995
t-Statistic	3.359	21.867	-2.702	0.218		
Confidence Level	99.914%	100.000%	99.278%	17.255%		
1990M01 2000M12						
Estimate	3.254%	0.93311	--	--	0.88000	1.377
t-Statistic	5.560	32.061	--	--		
Confidence Level	100.000%	100.000%	--	--		
Estimate	3.220%	1.23763	-0.32258	--	0.89200	2.082
t-Statistic	7.011	15.548	-4.005	--		
Confidence Level	100.000%	100.000%	99.990%	--		
Estimate	3.207%	1.18681	-0.13027	-0.15346	0.89382	1.988
t-Statistic	7.988	13.307	-0.918	-1.836		
Confidence Level	100.000%	100.000%	63.942%	93.125%		
2001M01 2020M09						
Estimate	0.861%	0.92365	--	--	0.83246	1.730
t-Statistic	2.719	41.368	--	--		
Confidence Level	99.296%	100.000%	--	--		
Estimate	0.826%	1.06961	-0.16121	--	0.83603	2.001
t-Statistic	3.061	17.111	-2.667	--		
Confidence Level	99.754%	100.000%	99.181%	--		
Estimate	0.838%	1.07975	-0.22859	0.06409	0.83600	2.027
t-Statistic	2.931	16.958	-2.493	1.024		
Confidence Level	99.629%	100.000%	98.665%	69.304%		

Real Ex Ante NDR Based on 5-Year Treasury Rate

	Long Run NDR	AR(1)	AR(2)	AR(3)	Adj R^2	DW
1990M01 2020M09					**# of Observations = 369**	
Estimate	1.601%	0.95069	--	--	0.90074	1.626
t-Statistic	3.102	54.268	--	--		
Confidence Level	99.793%	100.000%	--	--		
Estimate	1.574%	1.13534	-0.19508	--	0.90425	1.985
t-Statistic	3.641	22.708	-4.003	--		
Confidence Level	99.969%	100.000%	99.992%	--		
Estimate	1.577%	1.14019	-0.22327	0.02493	0.90405	1.995
t-Statistic	3.541	22.446	-2.939	0.469		
Confidence Level	99.955%	100.000%	99.650%	36.036%		
1990M01 2000M12					**# of Observations = 132**	
Estimate	3.067%	0.93982	--	--	0.88452	1.486
t-Statistic	4.859	29.745	--	--		
Confidence Level	100.000%	100.000%	--	--		
Estimate	2.965%	1.18940	-0.26562	--	0.89189	2.047
t-Statistic	5.960	13.220	-2.984	--		
Confidence Level	100.000%	100.000%	99.659%	--		
Estimate	2.930%	1.15624	-0.11807	-0.12392	0.89274	1.996
t-Statistic	6.586	12.625	-0.890	-1.425		
Confidence Level	100.000%	100.000%	62.504%	84.335%		
2001M01 2020M09					**# of Observations = 237**	
Estimate	0.925%	0.92331	--	--	0.83903	1.662
t-Statistic	2.184	36.509	--	--		
Confidence Level	97.006%	100.000%	--	--		
Estimate	0.886%	1.10239	-0.19605	--	0.84449	1.999
t-Statistic	2.506	17.375	-3.206	--		
Confidence Level	98.710%	100.000%	99.847%	--		
Estimate	0.893%	1.11230	-0.25373	0.05299	0.84425	2.021
t-Statistic	2.360	17.380	-2.685	0.780		
Confidence Level	98.090%	100.000%	99.223%	56.400%		

Appendix - Details Underlying the Autoregressive Model Estimates of the Long-Run NDRs (continued)

NDRs Based on the 10-Year Treasury Rate

	Long Run NDR	AR(1)	AR(2)	AR(3)	Adj R^2	DW	Long Run NDR	AR(1)	AR(2)	AR(3)	Adj R^2	DW
	Nominal NDR Based on 10-Year Treasury Rate						***Real Ex Ante NDR Based on 10-Year Treasury Rate***					
1990M01 2020M09												**# of Observations = 369**
Estimate	2.125%	0.95349	--	--	0.90846	1.654	2.029%	0.94428	--	--	0.88790	1.645
t-Statistic	4.559	62.531	--	--			4.476	51.344	--	--		
Confidence Level	99.999%	100.000%	--	--			99.999%	100.000%	--	--		
Estimate	2.127%	1.12410	-0.17908	--	0.91116	1.987	2.017%	1.11969	-0.18674	--	0.89150	1.980
t-Statistic	5.403	21.858	-3.583	--			5.299	23.087	-3.949	--		
Confidence Level	100.000%	100.000%	99.961%	--			100.000%	100.000%	99.991%	--		
Estimate	2.127%	1.12768	-0.20147	0.01992	0.91095	1.994	2.019%	1.12678	-0.22915	0.03804	0.89136	1.994
t-Statistic	5.295	21.301	-2.668	0.418			5.085	23.020	-3.097	0.714		
Confidence Level	100.000%	100.000%	99.203%	32.400%			100.000%	100.000%	99.789%	52.448%		
1990M01 2000M12												**# of Observations = 132**
Estimate	3.504%	0.93088	--	--	0.87656	1.438	3.340%	0.93656	--	--	0.88058	1.510
t-Statistic	6.310	32.279	--	--			5.795	29.564	--	--		
Confidence Level	100.000%	100.000%	--	--			100.000%	100.000%	--	--		
Estimate	3.494%	1.20513	-0.29081	--	0.88644	2.071	3.266%	1.17385	-0.25265	--	0.88743	2.035
t-Statistic	7.726	14.637	-3.551	--			7.088	13.121	-2.887	--		
Confidence Level	100.000%	100.000%	99.946%	--			100.000%	100.000%	99.543%	--		
Estimate	3.490%	1.16030	-0.10807	-0.14947	0.88821	2.001	3.242%	1.14655	-0.12715	-0.10636	0.88786	2.001
t-Statistic	8.765	13.094	-0.808	-1.858			7.732	12.457	-0.978	-1.265		
Confidence Level	100.000%	100.000%	57.919%	93.458%			100.000%	100.000%	66.991%	79.172%		
2001M01 2020M09												**# of Observations = 237**
Estimate	1.432%	0.89002	--	--	0.77149	1.728	1.420%	0.91183	--	--	0.81886	1.674
t-Statistic	6.381	33.881	--	--			3.873	33.449	--	--		
Confidence Level	100.000%	100.000%	--	--			99.986%	100.000%	--	--		
Estimate	1.419%	1.03923	-0.17079	--	0.77706	2.006	1.402%	1.08650	-0.19320	--	0.82478	1.999
t-Statistic	7.446	16.580	-2.787	--			4.548	17.416	-3.241	--		
Confidence Level	100.000%	100.000%	99.424%	--			99.999%	100.000%	99.863%	--		
Estimate	1.423%	1.04907	-0.23133	0.05925	0.77688	2.028	1.404%	1.09760	-0.25879	0.06123	0.82468	2.024
t-Statistic	7.088	16.349	-2.500	0.919			4.250	17.572	-2.781	0.893		
Confidence Level	100.000%	100.000%	98.688%	64.080%			99.997%	100.000%	99.414%	62.722%		