Stationarity Revisited, With a Twist

David G. Tucek
Value Economics, LLC
david.tucek@valueeconomics.com
314 434 8633

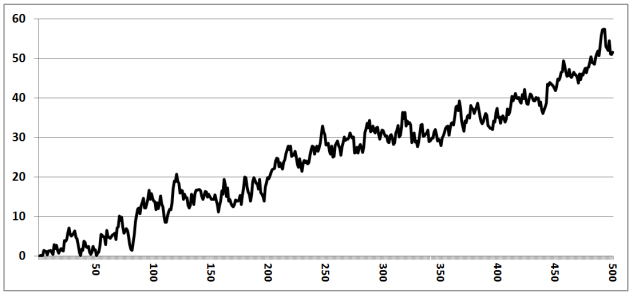
Why This Topic

- **Three Types of FEs**
  - **Those who learned little or no econometrics and times series analysis.**
  - **Those who learned pre-Granger/Newbold.**
  - **Those who learned post-Granger/Newbold.**

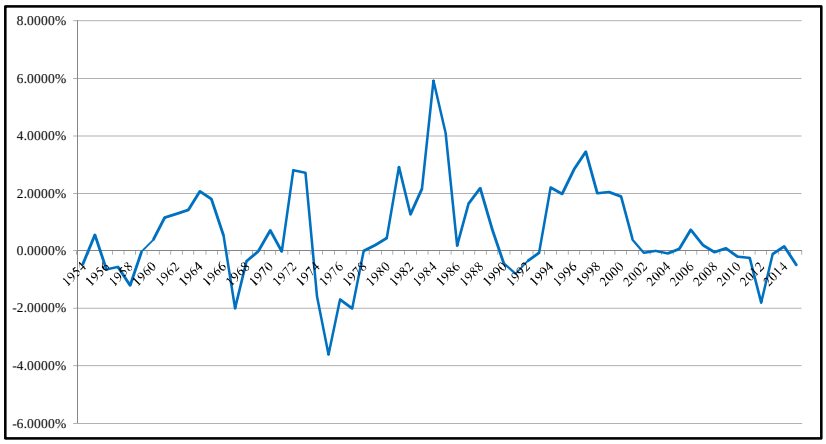
Spurious regression if regression variables are not stationary or cointegrated.

- **All three groups can benefit**
  - **Introduced to something new.**
  - **Should understand why you can't rely on the literature dealing with stationarity of NDRs as being definitive in a specific case.**
- **Dave can benefit**
  - **Presenting is thinking (to paraphrase Ireland).**
  - **Have I misunderstood or overlooked something?**

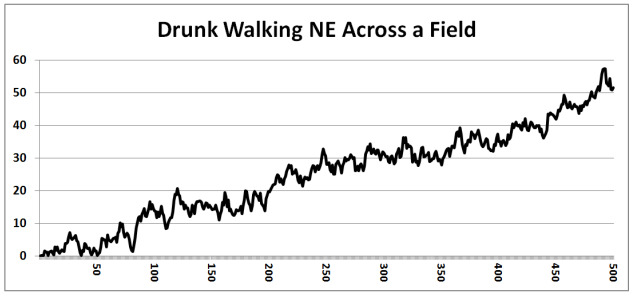
A Nonstationary Series



A Stationary Series

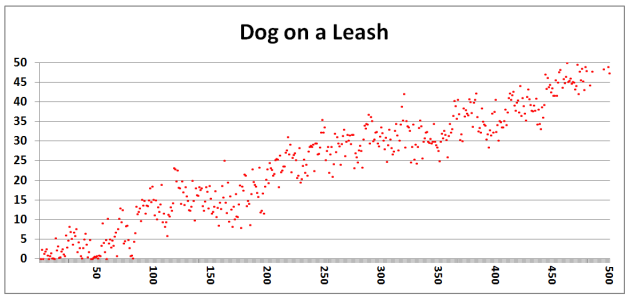


A Nonstationary Series

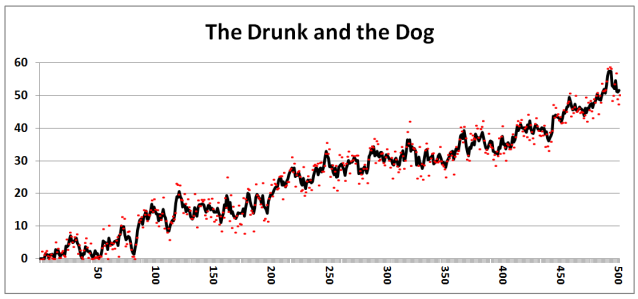


(Based on Murray, 1994)

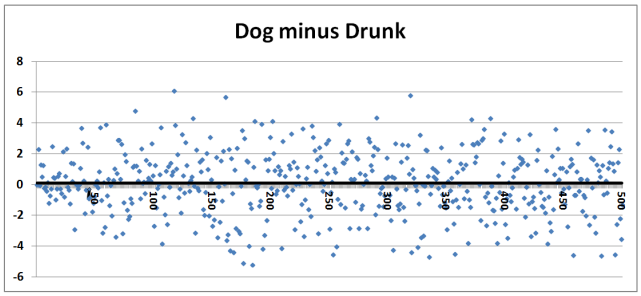
Another Nonstationary Series



Cointegrated Series
(By Design)

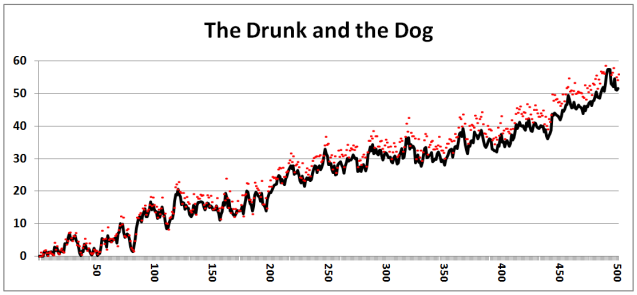


Difference is Stationary

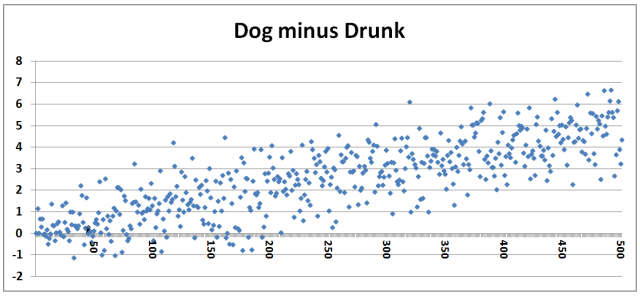


Suppose We Change the Design

- **Dog is not on a leash.**
- **Something towards the north that attracts the dog.**
- **Drunk looses interest in calling the dog as time passes.**



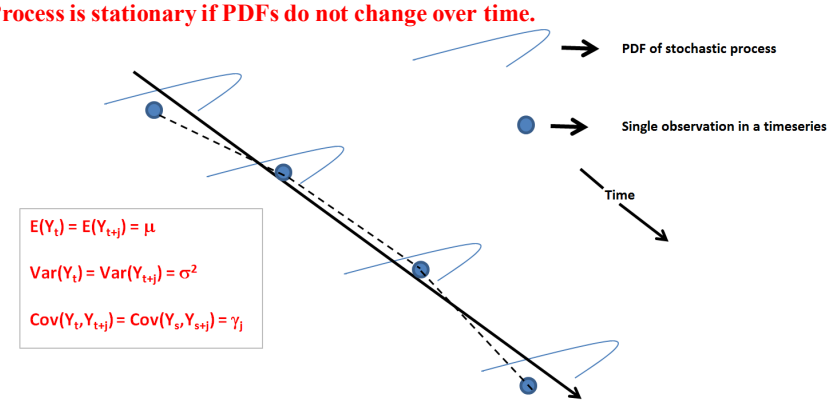
Difference is Not Stationary



Need to Test for Stationarity

Another Representation

Process is stationary if PDFs do not change over time.



We only have the observed timeseries to work with to reach decision on stationarity question.

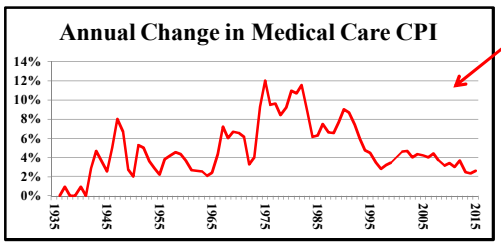
Testing for Stationarity

- **Look at a plot of the data.**
- **Formal tests for stationarity.**
- **Autocorrelogram.**
- **Estimate ρ in $Y_t = \alpha + \rho \cdot Y_{t-1} + \varepsilon_t$ and see whether $|\rho| < 1$ and by how much.**

Look at a Plot of the Data

- **Can be definitive, but often subject to judgment.**
- **Like looking at plot of regression residuals to detect autocorrelation or heteroscedasticity.**
  - **Risk of reducing decidion to a Rorschach test.**
  - **You see what you want to see.**

Stationary or nonstationary?



Formal Tests for Stationarity

- **Make some assumption about Y_t**

$$Y_t = \alpha + \rho \cdot Y_{t-1} + \sum_{j=1}^{k} \lambda_j \Delta Y_{t-j} + \varepsilon_t$$

Augmented Dickey-Fuller Equation

- **$|\rho| < 1$ ➔ stationary**
  $|\rho| = 1$ ➔ not stationary (unit root)
  $|\rho| > 1$ ➔ explosive

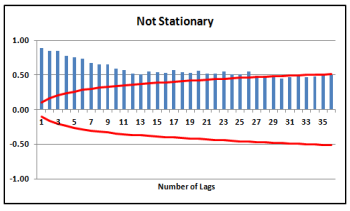
- **H_0: $|\rho| = 1$ (unit root) [not stationary]**
  H_1: $|\rho| < 1$ [stationary]

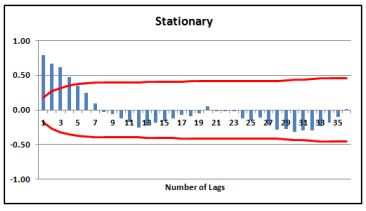
Problems with Formal Tests

- **Assumption about Y_t may be wrong.**
- **If true value of $|\rho|$ is close to but less than 1, tests may fail to reject the null hypothesis. (The tests are weak.)**
- **Failure to reject the null of a unit root does not mean you have proven the process is not stationary.**
  - **Type 1 Error: Reject the null hypothesis when it is in fact true.**
  - **Type 2 Error: Fail to reject the null when the alternative is true.**
  - **Reject the null hypothesis at a 95% level of confidence, you will be wrong (Type 1 error) 5% of the time.**
  - **If you do not reject the null hypothesis (and instead accept it) the probability of being wrong (Type 2 error) is <u>not</u> 5%.**
  - **Absence of evidence may be evidence of absence, but it is not <u>proof</u> of absence.**

Autocorrelogram

- **Plot of correlations between Y_t and Y_{t-1}, Y_{t-2}, . . .**
- **ρ_k = sample correlation between Y_t and Y_{t-k}**
- **The ρ_k decline to zero if the process is stationary.**





Estimating s_k, the Standard Error of the ρ_k

- $s^2_1 = 1/N$, where N is the sample size
- $s^2_k = (1 + 2\Sigma\rho^2_j)/N$, for k>1 where j=1, k-1
- 95% confidence interval $\approx \rho_k \pm 1.96 s_k$ (leaves 2.5% in each tail)

Note that the first two bullets have the variance on the left hand side, and that the confidence interval depends on the standard error (square root of the variance).

(See Box, Jenkins, *et al.*, 2015)

Side Note on Autocorrelogram

- Most commercial software packages (including SAS and eViews) calculate the correlogram based on the scaled sample autocovariances:

$$g_h = \frac{\sum_{t=h+1}^{T}(Y_t - \bar{Y})(Y_{t-h} - \bar{Y})}{\sum_{t=1}^{T}(Y_t - \bar{Y})^2}$$

where T is the total sample size and $\bar{Y}$ is the mean over the entire sample.

Side Note on Autocorrelogram

- **An alternative formula, based on the sample correlation between Y_t and Y_{t-h}, exists:**

$$r_h = \frac{\sum_{t=h+1}^{T}(Y_t - \bar{Y}_{h+1}^{T})(Y_{t-h} - \bar{Y}_{1}^{T-h})}{\sqrt{\{\sum_{t=h+1}^{T}(Y_t - \bar{Y}_{h+1}^{T})^2 \{\sum_{t=h+1}^{T}(Y_{t-h} - \bar{Y}_{1}^{T-h})^2\}}}$$

where $\bar{Y}_{v}^{T-w}$ is the sample mean of $Y_v, \ldots Y_{T-w}$.

- **When Y_t is stationary, the results of the two formulas are nearly identical. However, when Y_t is not stationary, the results from the two formulas can be very different and the first can incorrectly lead to the conclusion that Y_t is stationary.** (See Nielsen, 2006).

Estimate ρ in $Y_t = \alpha + \rho \cdot Y_{t-1} + \varepsilon_t$ and see whether $|\rho| < 1$ and by how much.

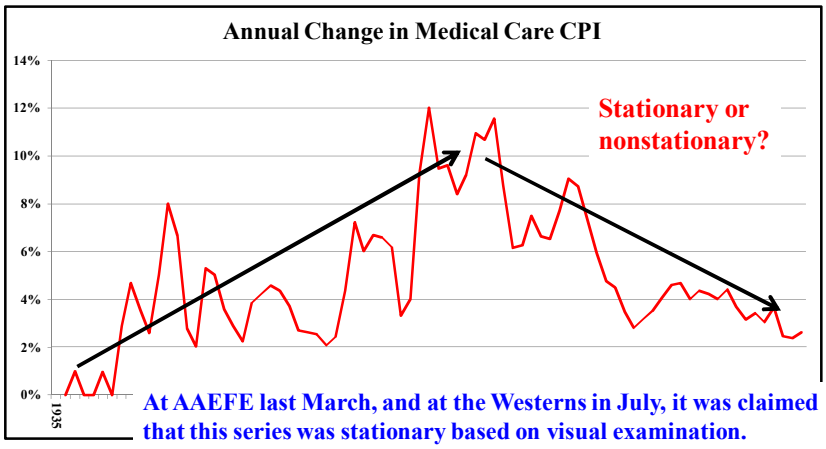
- **OLS estimate of ρ in $Y_t = \alpha + \rho \cdot Y_{t-1} + \varepsilon_t$ is biased, as is its estimated standard error.**
- **Correct bias in $\hat{\rho}$ as follows:**

  corrected $\hat{\rho} = [(N-1)/N-3)] \cdot \hat{\rho} + 1/(N-3)$
- **Correct bias in $s_{\hat{\rho}}$ as follows:**

  corrected $s_{\hat{\rho}} = \{[1-(\text{corrected } \hat{\rho})^2]/N - [1-14 \cdot (\text{corrected } \hat{\rho})^2]/N^2\}^{0.5}$
- **Calculate [1 - |corrected $\hat{\rho}$ |] ÷ corrected $s_{\hat{\rho}}$**

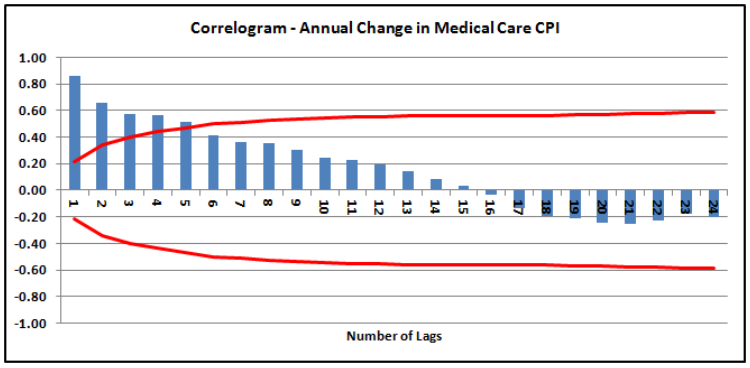
(See Orcutt and Winokur, 1969)

Example #1



At AAEFE last March, and at the Westerns in July, it was claimed that this series was stationary based on visual examination.

Example #1 - Autocorrelogram



Example #1 - Formal Tests

p-Values

- **Augmented Dickey-Fuller: 0.1521 (Reject null ≈ 85% level)**
- **Phillips-Perron**
  - **Bartlett's Kernel: 0.0830 (Reject null ≈ 92% level)**
  - **OLS AR Spectral: 0.1410 (Reject null ≈ 86% level)**

Example # 1

Estimate ρ in $Y_t = \alpha + \rho \cdot Y_{t-1} + \varepsilon_t$

- **$\hat{\rho}_{OLS} = 0.8433$; OLS estimate of $s_{\hat{\rho}} = 0.0578$.**
- **$\hat{\rho}_{OW} = 0.8787$; OW estimate of $s_{\hat{\rho}} = 0.0668$.**
- **Distance of $\hat{\rho}$ from 1, divided by standard error:**
  - **OLS estimate: 2.71**
  - **OW estimate: 1.82**

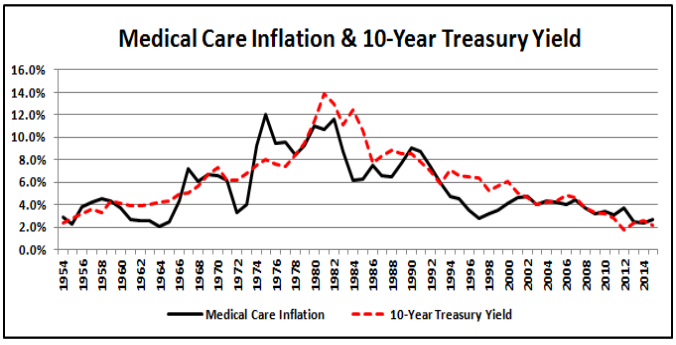
("OW" ➔ Orcutt/Winokur)

Example #1

IF there was a reason to believe that annual change in medical care CPI was stationary, the data support this belief.

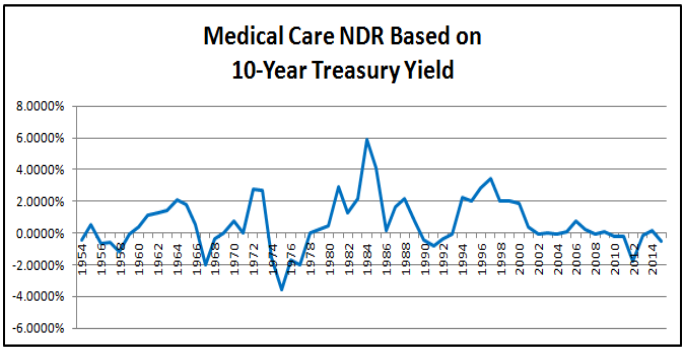
(Not a useful question to ask – relying on data starting in 1935 for an average growth rate is suspect.)

Example #2



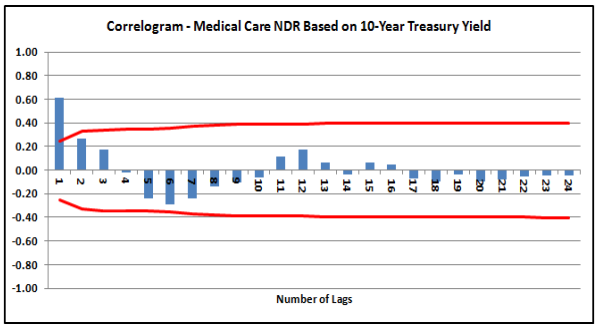
Like the drunk and the dog, the two series track each other – both are influenced by inflation. Plus, medical price changes are driven in part by overall economic growth and by the returns to capital and labor which are also related to interest rates.

Example #2



Still need to ask
"Is the NDR stationary?"

Example #2 - Autocorrelogram



Example #2 - Formal Tests

p-Values

- **Augmented Dickey-Fuller: 0.0051 (Reject null ≈ 99% level)**
- **Phillips-Perron**
  - **Bartlett's Kernel: 0.0035 (Reject null ≈ 99% level)**
  - **OLS AR Spectral: 0.0051 (Reject null ≈ 99% level)**

Example # 2
Estimate ρ in $Y_t = \alpha + \rho \cdot Y_{t-1} + \varepsilon_t$

- **$\hat{\rho}_{OLS} = 0.6099$; OLS estimate of $s_{\hat{\rho}} = 0.1032$.**
- **$\hat{\rho}_{OW} = 0.6482$; OW estimate of $s_{\hat{\rho}} = 0.1040$.**
- **Distance of $\hat{\rho}$ from 1, divided by standard error:**
  - **OLS estimate: 3.78**
  - **OW estimate: 3.38**

Example #2

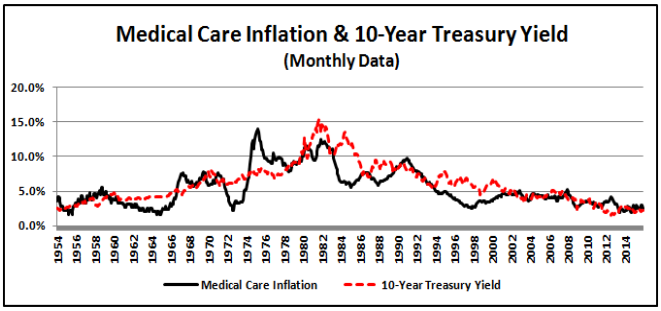
There is reason to believe that the annual change in the medical care CPI and the interest rate are related, and that the NDR is stationary. The data support this belief.

Note the subtle change: expectation first, data second.

" . . . the cointegrating relationship is not merely a statistical convenience with no behavioral content."

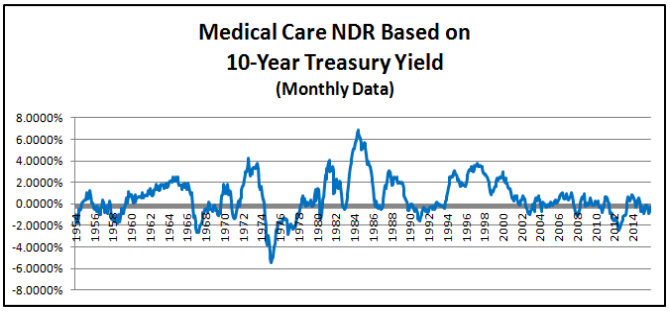
(Murray, 1994)

Example #2* - Monthly Data



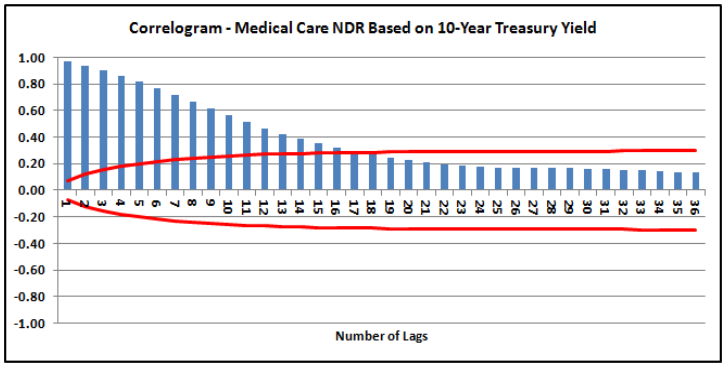
Like the drunk and the dog, and the annual data, the two monthly series track each other. This is expected for the same reasons.

Example #2* - Monthly Data



Still need to ask
"Is the NDR stationary?"

Example #2* - Autocorrelogram



Example #2* - Formal Tests

p-Values

- **Augmented Dickey-Fuller: 0.0004 (Reject null ≈ 99% level)**
- **Phillips-Perron**
  - **Bartlett's Kernel: 0.0007 (Reject null ≈ 99% level)**
  - **OLS AR Spectral: 1.25×10^{-6} (Reject null ≈ 99% level)**

Example # 2*

Estimate ρ in $Y_t = \alpha + \rho \cdot Y_{t-1} + \varepsilon_t$

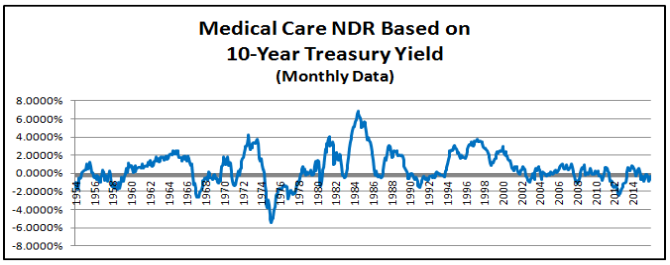
- **$\hat{\rho}_{OLS} = 0.9736$; OLS estimate of $s_{\hat{\rho}} = 0.0083$.**
- **$\hat{\rho}_{OW} = 0.9776$; OW estimate of $s_{\hat{\rho}} = 0.0091$.**
- **Distance of $\hat{\rho}$ from 1, divided by standard error:**
  - **OLS estimate: 3.17**
  - **OW estimate: 2.47**

At AAEFE and in Portland it was claimed the NDR could not be stationary because interest rates are a random walk.
Reinforces the need to look at the data and ask "Is the NDR stationary?"

Let's Read the Meter

- **Testing for stationarity involves more than just running a statistical test – don't click and run.**
- **Failure to reject the null hypothesis of a unit root doesn't mean the series is nonstationary – failing to reject the null is not the same as accepting it.**
- **Also, the statistical tests are weak. (If true value of $|\rho|$ is close to but less than 1, tests may fail to reject the null hypothesis.)**
- **We cannot generalize from published results – must look at data underlying each specific case. (Requires work.)**
- **Expectations come before data – the data may speak, but should only be listened to in response to a sensible question.**

The Twist
(or Maybe a "Taste")



- **Average NDR over entire period is 0.586%**
- **Average NDR since 2001 is -0.096%**
- **Most recent year's average is -0.483%**
- **What value should be used to discount future damages?**

Possible Approach

- **Estimate $Y_t = \alpha + \rho \cdot Y_{t-1}$ with an autoregressive error structure.**
- **Use this estimate to project Y_t into the future.**
- **Provides transition to a long-term average.**

Possible Approach

- **Estimate $Y_t = \alpha + \rho \cdot Y_{t-1}$ with an autoregressive error structure.**
- **Use this estimate to project Y_t into the future.**
- **Provides transition to a long-term average.**
- **Unanswered questions: What sample period to base estimate on?**

  What error structure to use?

Some Illustrative Results

Estimates based on 1954-2015

	Intercept	NDR(-1)	AR(1)	AR(3)	AR(4)	AR(12)	Adjusted R-Squared	Long-Run NDR	Date Long-Run Achieved
Estimate	0.000244	0.959789	0.214332				0.9506	0.61%	2036M04
t-Statistic	1.24	94.81	6.32						
Estimate	0.000141	0.976680	0.206442			-0.315909	0.9556	0.61%	2045M02
t-Statistic	1.02	122.86	6.99			-11.09			
Estimate	0.000168	0.971976	0.209012	0.097565		-0.315397	0.9560	0.60%	2042M02
t-Statistic	1.10	108.07	7.02	3.25		-11.10			
Estimate	0.000183	0.969379	0.206048	0.088312	0.056484	-0.313379	0.9561	0.60%	2040M10
t-Statistic	1.14	100.68	6.89	2.84	1.93	-11.08			

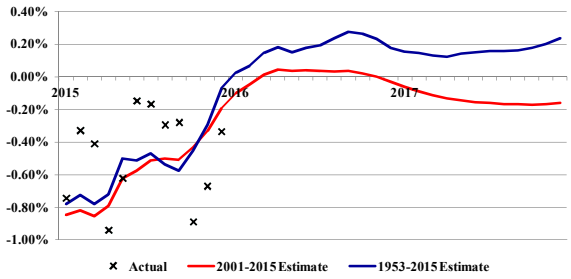
Some Illustrative Results

Estimates based on 2001-2015

	Intercept	NDR(-1)	AR(1)	AR(3)	AR(4)	AR(12)	Adjusted R-Squared	Long-Run NDR	Date Long-Run Achieved
Estimate	-0.000194	0.843636	0.304114				0.8446	-0.12%	2018M10
t-Statistic	-0.63	14.37	2.96						
Estimate	-0.000179	0.866342	0.278699			-0.195871	0.8502	-0.13%	2020M12
t-Statistic	-0.75	16.64	2.91			-2.67			
Estimate	-0.000278	0.762334	0.384576	0.190055		-0.169700	0.8534	-0.12%	2021M12
t-Statistic	-0.75	5.28	2.19	2.06		-2.36			

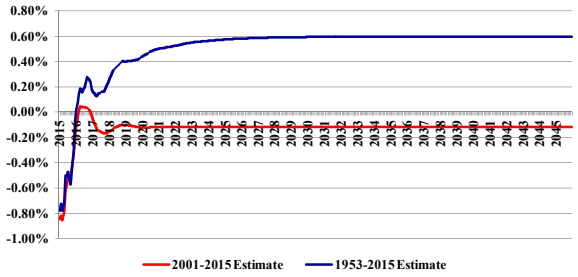
Some Illustrative Results

Actual & Projected NDR



Some Illustrative Results

Projected NDR



Some Illustrative Results

- **Error structure does not affect LR NDR.**
- **Choice of estimation period makes a big difference in LR NDR.**
- **The data alone cannot drive decision on estimation period.**
- **FE must decide whether future structure of economy makes a difference and, if it does, whether it has changed.**
- **Come to AAEFE for more on this topic.**

References

- **Box, George EP, et al. *Time series analysis: forecasting and control*. John Wiley & Sons, 2015.**
- **Granger, Clive WJ, and Paul Newbold. "Spurious regressions in econometrics." *Journal of econometrics* 2.2 (1974): 111-120.**
- **Murray, Michael P. "A drunk and her dog: an illustration of cointegration and error correction." *The American Statistician* 48.1 (1994): 37-39.**
- **Nielsen, Bent. "Correlograms for non-stationary autoregressions." *Journal of the Royal Statistical Society: Series B (Statistical Methodology)* 68.4 (2006): 707-720.**
- **Orcutt, Guy H., and Herbert S. Winokur Jr. "First order autoregression: inference, estimation, and prediction." *Econometrica: Journal of the Econometric Society* (1969): 1-14.**

Something for Everyone

Rene Descartes walks into a bar.

The bartender asks, “Can I get you a drink?”

Descartes replies, “I think not.”

. . . and he disappears.