Yet Another Net Discount Rate Paper

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Abstract

This paper presents the results of various unit root and stationarity tests for both *ex post* and *ex ante* measures of the net discount rate for wage growth. The analysis covers various subperiods from January 1982 to December 2012. The two NDR measures are both based on 10-year Treasury rates and on the year-earlier growth in monthly average weekly earnings for production and nonsupervisory workers reported by the Bureau of Labor Statistics. Implications for practicing forensic economists are presented.

Introduction

The question of whether or not net discount rates (NDRs) are stationary has long been debated among forensic economists. While the first paper on this topic published in *The Journal of Forensic Economics* was written by Nowak in 1991, the issue had been addressed much earlier elsewhere. For example, Hosek (1982) concluded that the NDR was not stationary while Franz (1978) concluded that earnings growth and interest rates were closely linked. Since then, the stationarity question has remained unresolved: of fourteen studies surveyed by Payne (2007), five concluded that the NDR was stationary, four concluded that the NDR was stationary with a break, three concluded it was not stationary and two had mixed findings on the issue.

This paper provides yet another contribution to this ongoing inquiry. The analysis covers various subperiods from January 1982 to December 2012 for both *ex post* and *ex ante* measures of the NDR for wage growth. To the extent that the paper makes a contribution to the studies that have gone before, it is through the journey rather than the destination.

The Data

Two NDR measures are studied, both based on 10-year Treasury rates and on the year-earlier growth in monthly average weekly earnings for production and nonsupervisory workers reported by the Bureau of Labor Statistics. The first NDR measure is based on the geometric difference between the nominal 10-year Treasury rate, i, and the growth, g, from a year earlier in the average weekly earnings series: $NDR_1 = (1+i)/(1+g) - 1$. The second measure is calculated in the same way, except that the growth rate is replaced with the year-earlier growth in earnings deflated by the CPI-U, and i is replaced with a real rate, r, calculated as the geometric difference between i and the measure of 10-year <u>expected</u> inflation, p, published by the Federal Reserve Bank of Cleveland.[1] Algebraically, $r = (1+i)/(1+p) - 1$ and $NDR_2 = (1+r)/(1+g^*) - 1$, where g^* is the growth in real average weekly earnings from a year earlier.

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[1] This measure of expected inflation is based on nominal Treasury yields, survey inflation forecasts, and inflation swap rates. See Haubrich (2009) and Haubrich, *et al*. (2008). The *Appendix* presents a comparison of various alternative measures of an *ex ante* real rate.

NDR_1 is an *ex post* measure of the net discount rate because it depends entirely on past values of interest rates, wage growth and inflation. Because NDR_2 depends on expected inflation, it is an *ex ante* measure of the net discount rate. Note that if past inflation were substituted for expected inflation in the calculation of NDR_2, the resulting values would be identical to NDR_1.

The period studied starts in January 1982 (the first observation for the expected inflation measure used in the calculation of NDR_2) and ends with December 2012. In addition to the entire study period, four subperiods were examined: (1) January 1982 through December 1991; (2) January 1992 through December 2012; (3) January 1992 through December 2000; and (4) January 2001 through December 2012. The mean NDRs for the entire period and for each subperiod are shown in Table 1 below:

Table 1 – Mean Net Discount Rates

Time Period	NDR_1	NDR_2
1982 - 2012	3.02%	2.92%
1982 - 1991	5.62%	5.53%
1992 - 2012	1.79%	1.68%
1992 - 2000	2.97%	2.48%
2001 - 2012	0.90%	1.07%

As explained below, the dividing point between the first two subperiods corresponds to the end of the Federal Reserve's targeting of the M2 money supply. The dividing point between the last two subperiods was selected to encompass the two most recent recessions in the latter subperiod. This was done because the recovery in total employment for these two recessions differs markedly from the 10 previous recessions, as is shown in Figure 1. This figure depicts the employment gap – the period needed to recover the employment level experienced in the month before a recession starts – for each U.S. recession since 1945. The decline in total employment growth since 2001 is also evidenced by the trendline growth during the expansion following each recession, shown in Table 2 below:

Table 2 – Trendline Growth in Employment During Business Expansions

Expansion Period	Trendline Growth
Oct-1945 to Oct-1948	4.92%
Oct-1949 to June-1953	4.19%
May-1954 to July-1957	3.19%
April-1958 to March-1960	3.91%
Feb-1961 to Nov-1969	3.49%
Nov-1970 to Oct-1973	3.61%
March-1975 to Dec-1979	4.11%
July-1980 to June-1981	1.92%
Nov-1982 to June-1990	2.87%
Mar-1991 to Feb-2001	2.37%
Nov-2001 to Nov-2007	1.15%
June-2009 to Aug-2013	1.28%

The Stationarity Question

The monthly record of both NDR measures for the entire period is depicted in Figure 2. A casual examination of these two charts suggests that both measures exhibit a downward trend and are not stationary. Augmented Dickey Fuller (ADF) tests produce the following p-values:

Table 3 – ADF Test Results for Entire Period

NDR Measure	One-Sided ADF p-value
NDR_1	0.64803
NDR_2	0.47129

These values indicate that the null hypothesis that $\rho=1$ in

$$Y_t = \alpha + \rho \cdot Y_{t-1} + \sum_{j=1}^{k} \lambda_j \Delta Y_{t-j} + \varepsilon_t, \qquad (1)$$

where Y_t is the NDR at time t, cannot be rejected. In other words, the ADF tests support the conclusion that neither NDR is stationary.

This is not surprising. Gamber and Sorensen (1994) found that the NDR was stationary with a structural break occurring around October 1979 and January 1980. Gelles and Johnson (1996), attributed this break to a number of factors: (1) a decline in the power of unions; (2) an increase in the proportion of service industry jobs; (3) declining inflation; (4) the increase in the federal debt; (5) financial deregulation and (6) the change in Fed monetary policy in the early 1980's. Of these, (6) is perhaps the most relevant, given that the shift in Federal Reserve policy from targeting short-term interest rates to targeting money-supply growth began in October 1979 under the chairmanship of Paul Volcker. Though Chairman Volcker's efforts were unquestionably successful, the impact on interest rates and wage growth was asymmetrical. For the 12 months after October 1979, the 10-year Treasury rate averaged 11.09 percent, compared to the 9.04 percent for the prior 12 months – an increase of 205 basis points. The comparable change in earnings growth was a decrease of 93 basis points, resulting in a significant increase in the nominal net discount rate, NDR1. The monthly record of NDR1 before and after October 1979 is shown in Figure 3. The existence of a break sometime after October 1979 is unmistakable.

If the shift from targeting interest rates to targeting money supply growth was the driving force behind the break in the NDR, it is natural to ask what happened to the NDR after the Federal Reserve abandoned money supply targets as a policy tool. Freidman (2005) notes that the FOMC stopped setting a target for M1 growth in 1987, and in 1993 publicly acknowledged the downgrading of its M2 target. According to Freidman, this was "a change that most observers of U.S. monetary policy had already noticed well before then."

On the basis of Friedman's assessment, the 1982-to-2012 period of Figure 2 has been divided into the first two subperiods in Table 1 above. Figures 4 and 5 show the monthly record for NDR1 and NDR2 for each of these subperiods, respectively. Both NDR1 and NDR2 appear to be stationary for the earlier period. For the later period, both series seem to exhibit a slight downward trend, with NDR2 offering the best prospects for stationarity. Figures 6 and 7 present the monthly record for NDR1 and NDR2 for the last two subperiods shown in Table 1. Not surprisingly, Figure 6 exhibits the same pattern as seen in Figure 5: a slight downward trend with the real, ex ante NDR offering the best prospects for stationarity. In Figure 7, the slight downward trend is absent and both the nominal and real measures appear to be stationary.

Unit root tests were performed for all of the periods identified above. In addition to the ADF test, two Phillips Peron (PP) and one Kwiatkowski, Phillips, Schmidt and Shinn (KPSS) test were performed for each subperiod and for the entire study period. (The difference between the two PP tests lies in the technique to estimate the distribution of the error term in the PP test equation.) The results are presented in Table 4.

The KPSS tests in the rightmost column of Table 4 are based on the null hypothesis that the series in question is stationary, so that being unable to reject the null provides support for the conclusion that the NDR in question is stationary. As seen in Table 4, such support is not forthcoming except for the 1982 to 1991 subperiod. To the extent that being able to reject the KPSS null only at a 10 percent level of confidence is a more compelling case for stationarity than being able to reject it at a 5 or 1 percent confidence level, these results offer the best prospects for stationarity in two the post-1992 subperiods for the real, ex ante measure.2 The p-values resulting from the ADF and PP tests offer conflicting results, rejecting or supporting the conclusion of stationarity for the same NDR and subperiod. Note that this conflict exists not only between the ADF and PP tests, but also between the two alternative PP tests.

Table 5 presents a side-by-side comparison of the ADF and PP test results for the two NDRs in a slightly different way. In this table, the p-values have been replaced with a conclusion regarding the stationarity question: (1) a conclusion of "Not Stationary" corresponds to a p-value greater than 0.15; (2) a conclusion of "Stationary" corresponds to a p-value less than 0.05; (3) a conclusion of "Weak Support" corresponds to a p-value between 0.05 and 0.10; and (4) a conclusion of "Very Weak Support" corresponds to a p-value between 0.10 and 0.15.

With respect to the nominal, ex post NDR, the ADF tests support the conclusion of stationarity only for periods since 1992, and only then when the two subperiods are considered separately. By comparison, the PP tests using the Bartlett kernel to estimate the distribution of the error term in the PP test equation support the stationarity conclusion for all time periods, even for the entire study period in the face of an obvious downward trend. The PP tests using an autoregressive specification to estimate the distribution of the test equation's error term support the conclusion of stationarity only for periods since 1992.

With respect to the real, ex ante NDR, the ADF tests support the conclusion of stationarity only for the 1982-1991 period, and offer weak or very weak support for the stationarity conclusion for

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[2] The KPSS test results in Table 4 are based on an estimate the distribution of the error term in the KPSS test equation using Bartlett's kernel. Using an autoregressive technique results in the null being rejected at a 1 percent confidence level in every instance.

the subsequent periods. With one exception, the PP tests using the Bartlett kernel to estimate the distribution of the error term in the PP test equation support the stationarity conclusion for all time periods. The exception is the 1992-2000 subperiod, with a p-value of 0.06. The PP tests using an autoregressive specification to estimate the distribution of the test equation's error provide some support for the conclusion of stationarity for all periods, though it is weak for the entire period and very weak for the 1992-2000 subperiod.

Clearly, these results are not definitive. The PP tests using Bartlett's kernel give the most consistent results for both NDR measures, but also support the stationarity conclusion for the entire 1982-2012 period, even though visual inspection of the data make it clear that neither series is stationary over the entire period. The ADF results contradict these results for both NDRs except (perhaps) for the 2001-2012 period. The PP tests using an autoregressive specification to estimate the distribution of the test equation's error present a mixed bag of results: they are largely consistent with the ADF results for the entire period and the 1982-1991 subperiod, but are largely consistent with the other PP test results for the remaining periods.

Resolving the Conflict

Two approaches to resolving the conflict among the formal unit root tests were investigated. The first approach starts with the equation

$$Y_t = \alpha + \rho \cdot Y_{t-1} + \varepsilon_t, \qquad (2)$$

where Y_t is the NDR at time t.[3] Due to the presence of the lagged dependent variable, the ordinary least squares (OLS) estimate of ρ is biased but consistent if ε_t is not serially correlated. If ε_t is serially correlated, then the OLS estimate of ρ is biased and inconsistent; additionally the standard errors and resulting t-statistics cannot be relied upon. (Johnston, 1972). For $\rho>0$ the bias is positive, and for $\rho<0$ the bias is negative. Consequently, the OLS estimate of ρ will tend to understate its magnitude, leading to incorrect conclusions concerning the existence of a unit root.

If ε_t is not serially correlated, Orcutt and Winokur (1969) provide an estimate of ρ that corrects for the bias in the OLS estimate:

$$\rho_{OW} = (N/(N\text{-}3)) \cdot \rho_{OLS} + 1/(N\text{-}3) \qquad (3)$$

where ρ_{OW} is the corrected estimate of ρ, N is the number of observations and ρ_{OLS} is the OLS estimate of ρ. Additionally, they give an estimate of the variance of ρ_{OLS} as

$$\text{Var}(\rho_{OLS}) = (1\text{-}\rho^2)/(N\text{-}1) - (1 - 14\rho^2)/(N\text{-}1)^2 \qquad (4)$$

Since, for any random variable X, $\text{Var}(a + b\cdot X) = b^2\cdot\text{Var}(X)$, it follows that

$$\text{Var}(\rho_{OW}) = [N/(N\text{-}3)]^2 \cdot [(1\text{-}\rho^2)/(N\text{-}1) - (1 - 14\rho^2)/(N\text{-}1)^2] \qquad (5)$$

For $N>15$, the resulting values of $\text{Var}(\rho_{OLS})$ and $\text{Var}(\rho_{OW})$ decrease as $|\rho|$ increases. Accordingly, substituting ρ_{OLS} for ρ in (4) or (5) will tend to overestimate the calculated variance

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[3] This is the model specification underlying the original (non-augmented) Dickey-Fuller test and the PP tests. It is also the same as the equation underlying the ADF test, absent the $\sum_{j=1}^{k} \lambda_j \Delta Y_{t-j}$ term in (1).

and standard error. (That is, if we could use the true value of ρ to calculate the variance, the result would be lower, on average, than the result obtained by using ρ_{OLS}.)

When ε_t is serially correlated (*i.e.*, when $\varepsilon_t = \theta \cdot \varepsilon_{t-1} + u_t$), the probability limit of ρ_{OLS} is given by $(\rho + \theta)/(1 + \rho \cdot \theta)$. (Johnston, 1972). Setting ρ_{OLS} to this expression and solving for ρ suggests the following estimate of ρ when ε_t is serially correlated, given an estimate of θ:

$$\rho^* = (\theta - \rho_{OLS})/(\theta \cdot \rho_{OLS} - 1) \qquad (6)$$

In the results below, the correlation between the lagged residuals from the OLS estimate of (2) is used as an estimate of θ.

The above results were used to correct ρ_{OLS} for bias – if the resulting corrected estimate of ρ is less than 1 in absolute value, this correction provides support for the conclusion that the NDR is stationary. When ε_t in (2) is not serially correlated, it is possible to estimate how likely the corrected estimate of ρ is less than 1 in absolute value by estimating the variance of ρ_{OW} based on (5). An upper bound for a two-tail p-value for the null hypothesis that $|\rho|>1$ can be calculated based on the ratio of 1 minus $|\rho|$ divided by the square root of the variance and on Chebyshev's inequality. If we are willing to assume the distribution of ρ_{OW} is symmetric, dividing this by 2 gives a one-tail p-value for the null hypothesis that $|\rho|=1$. (When ε_t is serially correlated, no corresponding test based on the variance of ρ^* exists.)

The OLS estimates of ρ, along with the corrected estimates describe above, are presented in Table 6. The first column contains the OLS estimates of ρ, followed by Durbin's h-statistic and its associated p-value.[4] The next two columns contain the estimates of ρ corrected for bias (ρ_{OW} and ρ^*). The rightmost column contains the one-sided p-value for ρ_{OW} corresponding to the null hypothesis that $|\rho|=1$. As explained above, this p-value is based on the estimated variance of ρ_{OW} given by (5), and on Chebyshev's inequality and the assumption that the distribution of ρ_{OW} is symmetric. Entries that are in strike-out font signify values that are not applicable to the given time period, depending on whether Durbin's h-statistic indicates that the error term in (2) is or isn't autocorrelated.

At first blush, these results are encouraging: correcting the OLS estimate of ρ for bias results in values less than one in absolute value for all time periods. Unfortunately, this includes the entire time period: this exercise, like the PP tests in the middle of Table 5, support the conclusion of stationarity even when visual inspection of the data strongly suggests otherwise. Moreover, this approach to resolving the conflict among the formal unit root tests only partially addresses the possibility that, due to sample variation, the estimated value of ρ may be less than 1 even when ρ equals 1. While the p-values in the rightmost column provide some information on this aspect of the problem, they should not be viewed as having the same legitimacy of a statistical test based on a known sampling distribution. The importance of this precautionary observation increases with the magnitude of ρ, since we are interested in the significance of the difference between 1 and $|\rho|$. Consequently, these results offer the strongest support for the conclusion of stationarity for the 2001-to-2012 period, i.e., for the estimates of ρ furthest from 1.

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[4] Durbin's h-statistic is used to test for the presence of serially correlated error terms in an autoregressive model like (2). See Durbin (1970).

The second approach to resolving the conflict among the formal unit root tests involves examining the data in a different way, by calculating the correlogram for each NDR and for each time period.[5] Nielsen (2006) notes that most commercial software packages (including SAS and EViews) calculate the correlogram based on the scaled sample autocovariances:

$$g_h = \frac{\sum_{t=h+1}^{T}(Y_t - \bar{Y})(Y_{t-h} - \bar{Y})}{\sum_{t=1}^{T}(Y_t - \bar{Y})^2} \qquad (7)$$

where T is the total sample size and $\bar{Y}$ is the mean over the entire sample.

Nielsen notes that an alternative formula, based on the sample correlation betweenY_t and Y_{t-h}, exists:

$$r_h = \frac{\sum_{t=h+1}^{T}(Y_t - \bar{Y}_{h+1}^{T})(Y_{t-h} - \bar{Y}_1^{T-h})}{\sqrt{\{\sum_{t=h+1}^{T}(Y_t - \bar{Y}_{h+1}^{T})^2\,\{\sum_{t=h+1}^{T}(Y_{t-h} - \bar{Y}_1^{T-h})^2\}}} \qquad (8)$$

where $\bar{Y}_v^{T-w}$ is the sample mean of $Y_v, \ldots\ Y_{T-w}$.

When Y_t is stationary, the results of formulas (7) and (8) are nearly identical. However, when Y_t is not stationary, the results from the two formulas can be very different. Using previously studied data for four economic times series, Nielsen demonstrates that the conclusions reached on the basis of (7) and (8) can sometimes agree and, at other times, disagree. In particular, (7) resulted in the conclusion of stationarity for a series (the log of monthly prices during a period of

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[5] If the sample autocorrelations decay to zero, then the time series in question can be viewed as stationary. See Box and Jenkins (1971), Nielsen (2006) and Enders (2010) for examples of autocorrelation functions for stationary and non-stationary series.

Yugoslavian hyperinflation) believed to be an autoregressive process with a unit root and an explosive root. By comparison, the correlogram produced by (8) indicated persistently high values for r_h, consistent with the conclusion of non-stationarity. Because the basic issue addressed in this paper deciding whether the NDRs are stationary or non-stationary, the correlograms presented below are based on (8).

These results are presented in Figure 8: the values of r_h are represented by the vertical bars, while the solid lines correspond to a 95 percent confidence band based Bartlett's formula for the variance of the estimated autocorrelation coefficient. (Box and Jenkins, 1971). The correlograms decay to zero for all subperiods, but not for the entire 1982-2012 period. The results depicted in Figure 8 are summarized in Table 7. This table presents the number of lags with sample autocorrelation coefficients that are significantly different from zero at a 95 percent level of confidence. For the entire study period, these results clearly indicate that both NDR measures are not stationary, consistent with the visual inspection of Figure 2. For the subperiods, the greatest significant lag is 9 months with one exception, the 1992-2012 subperiod for the real, *ex ante* NDR. For this subperiod and this NDR, the sample autocorrelation coefficients are significantly greater than zero for 11 months, with an additional significant coefficient at a lag of 14 months. Inspection of correlogram (third panel on the right-hand side of Figure 8) reveals spikes in the sample correlation coefficients at months 17, 20, 23, 26, 29, 32 and 35, suggesting a weak seasonal effect. Nevertheless, the correlogram does decay to zero and supports the conclusion of stationarity.

Summary and Analysis of Results

Table 8 summarizes the above results in terms of whether or not they support the conclusion that each NDR measure is stationary. For the entire period, visual inspection, the ADF and KPSS unit root tests, and the correlogram analysis consistently agree that neither NDR measure is stationary over the entire period. The two PP tests and the autoregressive coefficient corrected for bias present conflicting, or at least mixed, results. Reviewing the remaining rows of Table 8, the strongest case for stationarity is made for the real, *ex ante* NDR measure for both the 1982-1991 and 2000-2012 subperiods: these are the only two subperiods for which each metric offers at least weak support for the stationarity conclusion. Setting aside the KPSS results, the next strongest case is made for the nominal, *ex post* NDR for the 2001-2012 subperiod followed by the real, *ex ante*, NDR for the 1992-2012 and 1992-2000 subperiods.

The contradictory results for the nominal, ex post NDR for the remaining subperiods are due to the conflicts among the formal unit root tests. For the 1981-1991 and 1992-2012 subperiods, the ADF tests are directly contradicted by the PP tests using the Bartlett kernel to estimate the distribution of the error term in the PP test equation. The KPSS and remaining PP test results are split, and do not agree with either of the first two tests, or with each other.

What is to be made of these diverse results and, more important, what weight should one give to each analytical method? To answer these questions, it is useful to understand the possible reasons the formal unit root tests fail to give consistent results for the same time series observations.

The first, and possibly most important, reason is that all of the unit root tests rely on an assumed specification of the underlying time series process, and/or on assumptions concerning the form of the accompanying error term. The ADF test equation is based on (1), and on the assumption that ε_t is IID with zero mean and constant variance. The PP test relies on (2), but permits ε_t to be autocorrelated and heteroscedastic. The KPPS test is based on $Y_t = r_t + \varepsilon_t$ where ε_t is stationary and $r_t = r_{t-1} + \mu_t$ with μ_t being IID zero mean and constant variance.[6] (The null for the KPSS test is that $Var(\mu_t)=0$. See Kwiatkowski, *et al.* (1992)). Failure to reject the null in the ADF and PP tests, or rejecting the null in the KPSS test, does not necessarily mean the series in question is not stationary – it could mean that the underlying model specification or the assumptions concerning the concerning the form of the accompanying error term, or both, are wrong. Indeed, Box and Jenkins (1971) have noted that time series are most often best represented (in terms of parsimony in the number of model parameters) by an autoregressive, moving average (ARMA) process rather than just a pure autoregressive or pure moving average process. None of the formal unit root tests discussed above rely on an ARMA specification of the underlying process.

A second reason why the formal unit root tests fail to give consistent results is that they are all based on models in which the differencing parameter is an integer. That is, the series in question is tested to see if it is I(0), (integrated of order zero), I(1) (integrated of order one), and so on. If it is not, then it is concluded that the series is not stationary. However, Clark, *et al.* (2008) note that, in estimating damages, what we are interested in is the property of *mean reversion*, and that the class of fractionally integrated process encompasses both stationary and non-stationary series that are mean reverting. Thus, it is possible that the formal unit root tests will fail to support the

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[6] Like (1) and (2), this equation corresponds to the level-stationary case. Tests for the trend-stationary case add a deterministic trend to the model specification.

stationarity conclusion either because they do not consider fractionally integrated process or because the series in question is non-stationary but nevertheless mean reverting.

Finally, DeJong, *et al*. (1992) have shown that Dickey-Fuller type tests have low power when the autoregressive process is persistent. That is, for large values of ρ that are less than 1 in absolute value (say, for $0.85 < |\rho| < 1$), the tests may fail to reject the null that $|\rho|=1$. This presents us with the paradox that when the historical mean of the NDR is a good predictor of future values because the autoregressive process is persistent, we are likely to fail to reject the null hypothesis of a unit root.

Do these weaknesses mean that any of the other metrics presented in Table 8 are to be preferred over the formal unit root tests? For visual inspection and the corrected autoregressive coefficient the answer is most likely "No." Even though visual inspection of the data suffers from none of the formal unit root tests' limitations, it is subject to interpretation bias. For example, in the results presented above, the subperiods starting in 1992 were characterized as having a "Slight Trend" for the nominal, *ex post* NDR and "Less of a Trend" for the real, *ex ante* NDR. The determination of what constitutes "slight" and "less of" is necessarily subjective.

The estimate of the autoregressive coefficient corrected for bias is probably the least informative with respect to the issue of stationarity: the corrected estimate alone only partially addresses the possibility that, due to sample variation, the estimated value of |ρ| may be less than 1 even when |ρ| equals or exceeds 1. Additionally, this metric depends on the specification of a first-order autoregressive equation as the process underlying the time series. Even so, the magnitude of the corrected coefficient estimate can provide guidance on how much weight to assign to the rejection of the null hypothesis under the ADF or PP tests: rejecting the null when the corrected estimate of |ρ| is large is not as (subjectively) significant as rejecting the null when the corrected estimate is small.

The correlogram analysis is perhaps the most useful analytical tool presented above. It does not rely on a pure autoregressive or moving average specification of the underlying time series process, instead only assuming that the process is stationary. Failure of the sample autocorrelations to decay to zero is a strong indication that this assumption is not valid. Moreover, the question of whether the sample autocorrelations decay to zero is not subjective, since deciding whether or not a value falls inside or outside of the confidence interval is a straightforward matter.

Implications for Practicing Forensic Economists

The analysis presented above results in several important implications for practicing forensic economists. First, and perhaps the most important, is that simply claiming that net discount rates have been shown to be non-stationary is, at best, disingenuous. The studies reviewed by Payne, as well as the results reported here, show that this is not the case. More to the point, failing to reject the null hypothesis of a unit root does not mean the NDR in question is not stationary. Several other possible explanations exist: (1) the series could be a fractionally integrated process that is stationary; (2) either the assumed time series process underlying the NDR and/or the assumptions regarding the error term could be inadequate; or (3) the NDR could be generated by a stationary, persistent, autogressive process with a value of |ρ| less than but close to one.

Finally, the NDR series could be nonstationary but mean reverting, which is the property that practicing forensic economists are ultimately concerned with.

Second, a related and nearly as important implication is that a forensic economist advocating a specific NDR (or a specific combination of a historical growth rate and discount rate) is on shaky ground unless the stationarity question is addressed. This is only “nearly as important” because the stationarity question will only become an issue if the opposing economist raises it. Still, not investigating the question before preparing your report runs the risk of having to admit you don’t know the answer to the question. or that the answer is that the series is not stationary.

Third, just as it is not sufficient to rely on some vague reference to the literature to counter the use of an (explicit or implicit) historical NDR, it is not sufficient to rely on only one or two unit root tests to support the conclusion of stationarity. Moreover, the above results make clear that actually looking at the data (as opposed to just pouring it into a software package) is needed to come to an objective conclusion concerning stationarity. In particular, an examination of the (correctly calculated) sample correlogram is called for.

Fourth, forensic economists who rely on historical averages should be cognizant not only of the structural break that Gamber and Sorensen identified as occurring in late 1979 or early 1980, but also of the break that apparently occurred in the early 1990’s, along with the break or shift that occurred in the 2000-2001 timeframe.[7] This latter shift is supported not only by the decline in

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[7] While 1979-1980 is probably adequately modeled as a sharp break given the abruptness of the change in monetary policy, the subsequent ‘break” in the early 1990’s may be better described as a gradual shift, since the FOMC did not abandon the targeting of monetary aggregates all at once. Enders and Lee (2102) suggest a test for an unknown number of endogenous breaks or other nonlinearities; this is an area for further research.

employment growth noted earlier, but also by the change in the relationship between the nominal, *ex post* NDR and the real, *ex ante* measure. Returning to Table 1, it is seen that NDR_1 is greater than NDR_2 for all but the 2001-2012 subperiod. This reversal, along with the sharp decline in each measure compared to its 1992-2000 average suggests that something happened in this timeframe.[8]

Finally, and somewhat unexpectedly, the above results have implications for advocates of the use of current interest rates as well. Both the quantitative findings presented above, and the studies reviewed by Payne, offer evidence of a link between nominal (or real) interest rates and nominal (or real) wage growth. Moreover, the link between interest rates and economic growth, and the impact of expectations on financial markets, are well-established economic tenets.[9] For example, Ben Bernanke has acknowledged that in the long term, real rates are determined by expected return to capital investments and the underlying strength of the economy, and that today's low real interest rates reflect the weakness of the recovery in advanced economies as well as some downgrading of longer-term growth prospects. Bernanke (2013).

At a minimum, this means that advocates of using current rates to discount future losses cannot rely on average historical growth rates to project those losses unless they turn a blind eye to the link between current interest rates and expected growth, or unless they can establish that the

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[8] One such factor may be the end of what John Taylor has called the "Long Boom" -- a period of economic expansion starting in November 1982 and which was interrupted only by the relatively short 1990 recession. Taylor attributes the existence and the end of the Long Boom to changes in monetary policy. See Taylor (1998) and (2010).

[9] For example, the Keynsian IS-LM model illustrates the relationship between (short-term) interest rates and output, while the expectations theory of the term structure of interest rates shows the financial markets are forward-looking.

historical average equals today's expected growth rate. It is impossible to do the latter – there is no market for wage growth, so the market's expected value cannot be observed. While some forensic economists try to resolve this shortcoming by combining current rates with a public (or purchased) forecast of wage growth, this solution founders as well. Such forecasts are demonstrably stale on the date they are published and, even if they were not, there is no guarantee that they consider all of the information reflected in the unobserved current market expectations, or even assigned that information the same weight as does the market. The best that can be hoped for is that the relationship between wage growth and interest rates is stationary, so that use of historical averages can be relied on.

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Appendix

The real, *ex ante* interest rate presented above is but one possible candidate for such a measure. This *Appendix* compares the real rate based on the estimate of expected inflation published by the Federal Reserve Bank of Cleveland with two alternatives. The first of these is obvious – the 10-year rate on Treasury Inflation Protected Securities (10-year TIPS). The second alternative is calculated in the same way as the rate used in the paper, with the expected inflation measure based on the forecasted 10-year inflation rate found in the *Survey of Professional Forecasters* (SPF) published by the Federal Reserve Bank of Philadelphia.

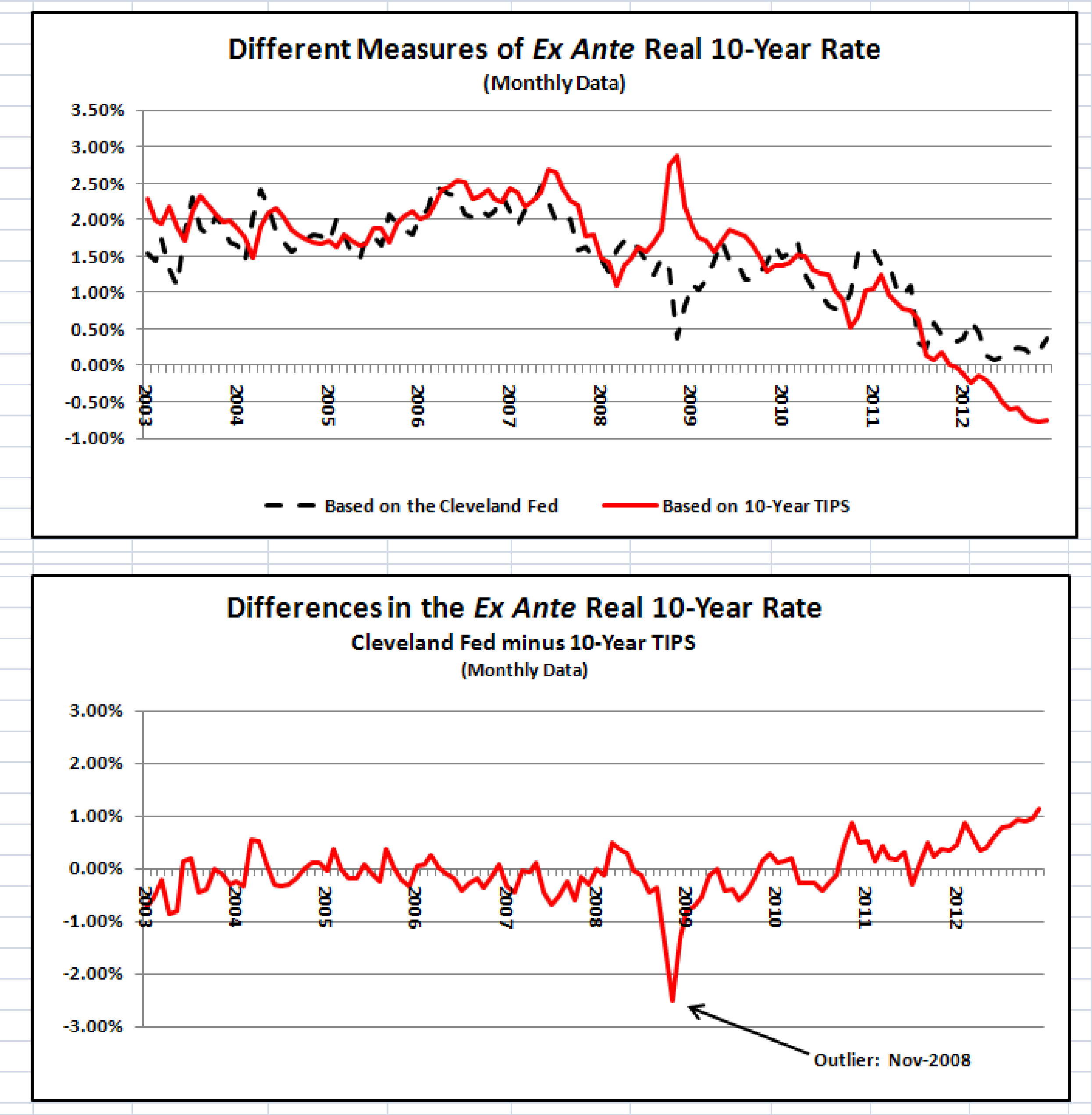
Neither of these series cover as long a time period as does the real interest based on the Cleveland Fed's expected inflation measure. Monthly data for the 10-year TIPS rate are available starting with January 2003. The SPF forecasts are published four times per year in February, May, August and November and are available starting with November 1991. Nevertheless, it is instructive to compare each of the alternatives with the real interest based on the Cleveland Fed's expected inflation measure for the time periods they share in common.

Figure A1 presents the comparison with the 10-year TIPS rate. The first panel in this figure plots both real interest rate measures; the second plots their difference. Both measures roughly track together and, except for the November 2008 observation, the differences fluctuate around zero. The increase in the difference during 2012 reflects an increase in the spread between the two expected inflation measures which has since subsided. This can be attributed to the effects of the Federal Reserve's initiation of QE3 and subsequent expectations of a tapering of their bond purchases, a decline in real rates due to expectations of slower economic growth, or both. The distribution of the differences is shown in Figure A2. Absent the November 2008 outlier, the distribution is equivalent to a normal distribution with a zero mean and the sample standard deviation.

Counterparts to Figures A1 and A2 appear in Figures A3 and A4, respectively, for the comparison with the real rate based on the SPF inflation forecast. This series was constructed based on the geometric difference between the nominal 10-year Treasury rate in the month before the *Survey*, and the survey forecast of inflation over the next 10 years. Again, both measures roughly track each other and the distribution of the differences is equivalent to a normal distribution with a zero mean and the sample standard deviation.

Figure A1

Comparison of Real Rate Based on Cleveland Fed's Expected Inflation with 10-Year TIPS Rate

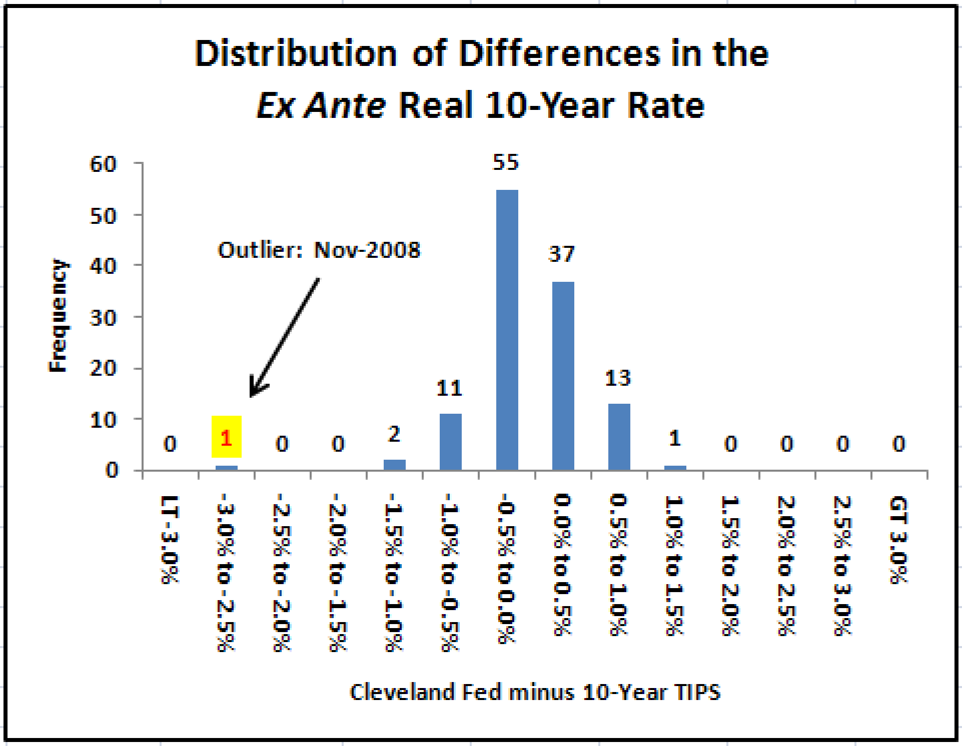


Different Measures of *Ex Ante* Real 10-Year Rate
(Monthly Data)

Differences in the *Ex Ante* Real 10-Year Rate
Cleveland Fed minus 10-Year TIPS
(Monthly Data)

Figure A2

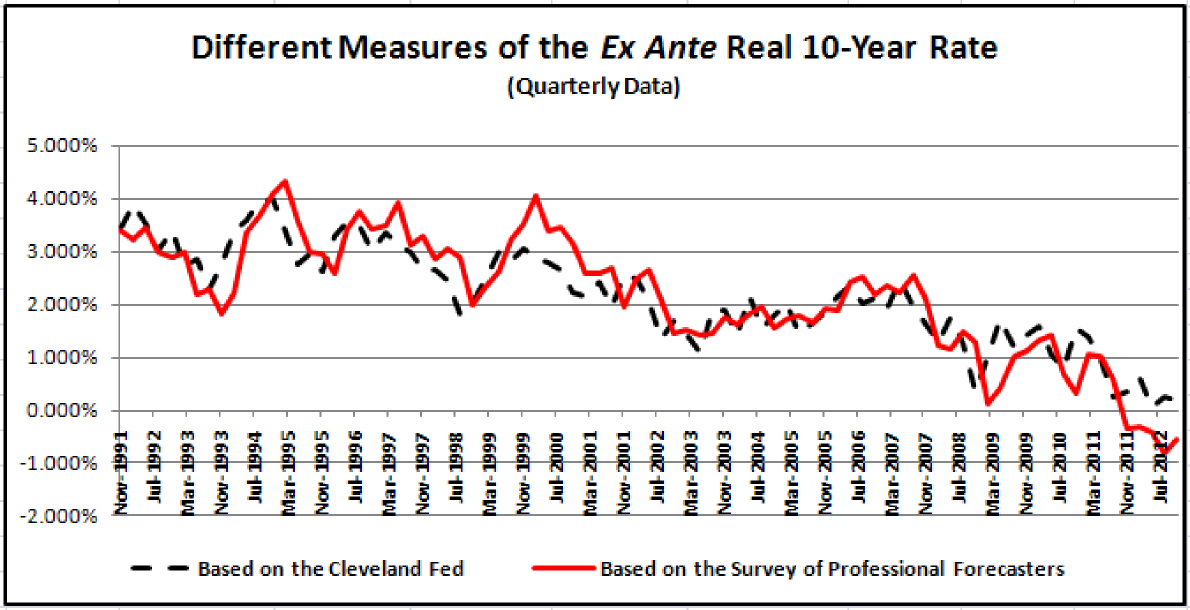
Distribution of Differences Between Real Rate Based on Cleveland Fed's Expected Inflation with 10-Year TIPS Rate



Sample Mean:	-0.053%	(with outlier)
Standard Deviation:	0.505%	'' ''
Sample Mean:	-0.033%	(without outlier)
Standard Deviation:	0.453%	'' ''

Figure A3

Comparison of Real Rate Based on Cleveland Fed's Expected Inflation with Rate Based on 10-Year Inflation Forecast From the *Survey of Professional Forecasters*



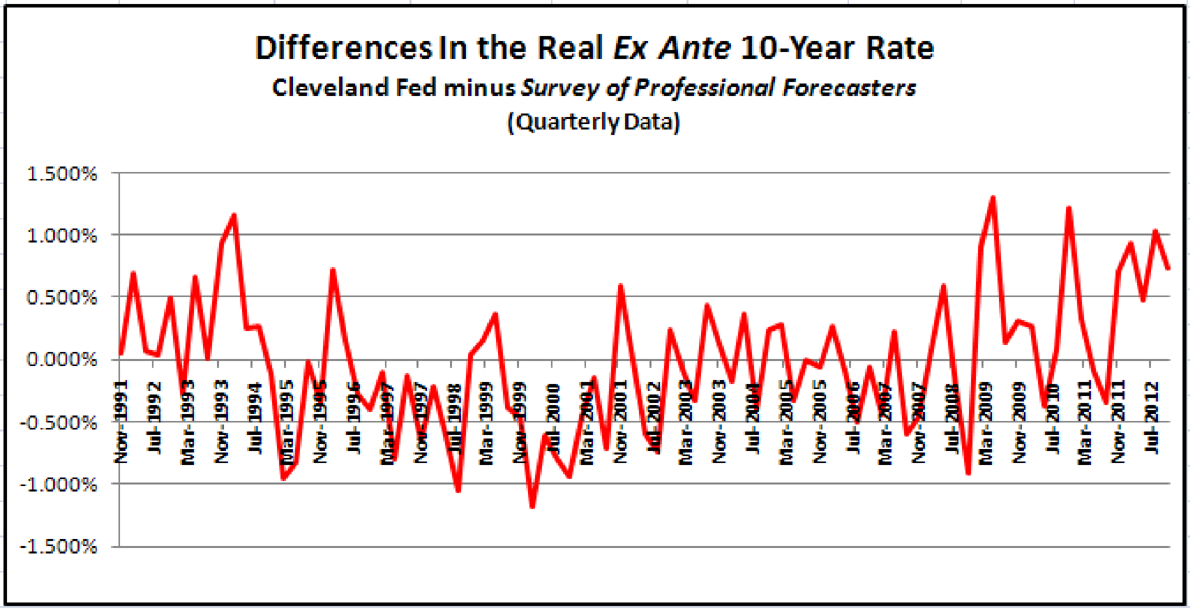
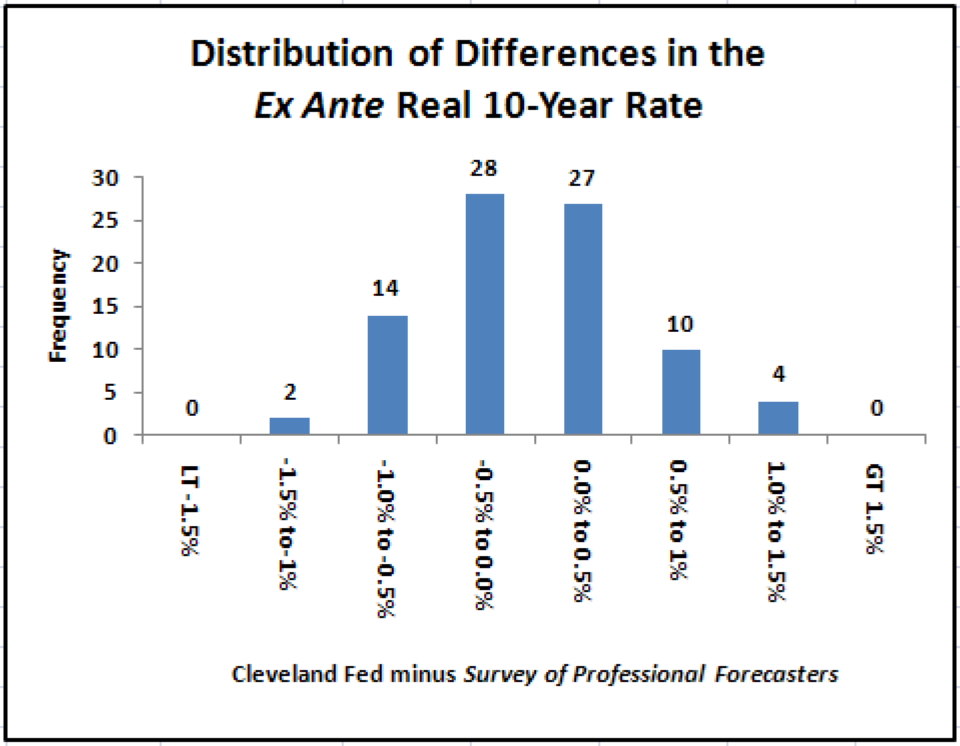


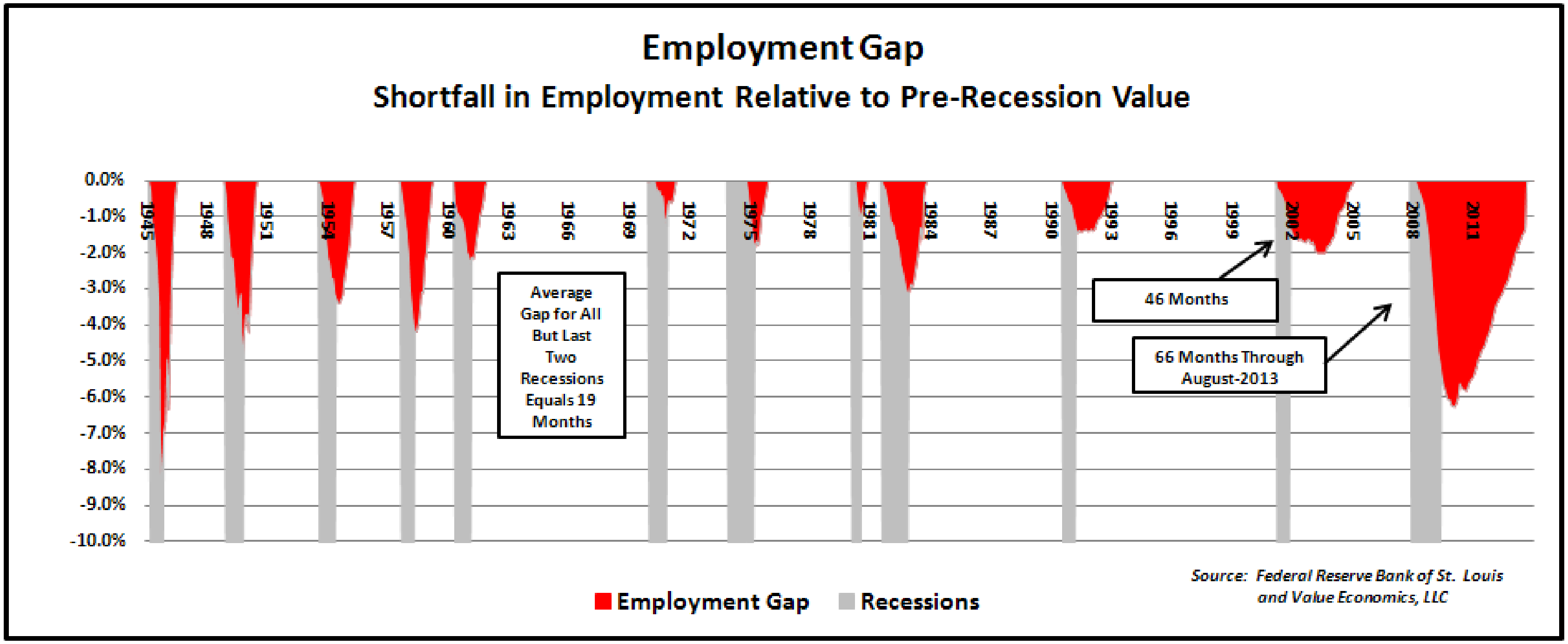
Figure A4

Distribution of Differences Between Real Rate Based on Cleveland Fed's Expected Inflation with Rate Based on 10-Year Inflation Forecast From the *Survey of Professional Forecasters*



Sample Mean:	-0.015%
Standard Deviation:	0.552%

Figure 1
The Employment Gap
Total Nonfarm Payroll Employment
(Monthly Data)



Employment Gap

Shortfall in Employment Relative to Pre-Recession Value

Average Gap for All But Last Two Recessions Equals 19 Months

46 Months

66 Months Through August-2013

Source: Federal Reserve Bank of St. Louis and Value Economics, LLC

Figure 2

Nominal (NDR_1) and Real (NDR_2) Net Discount Rates

(Monthly Data)

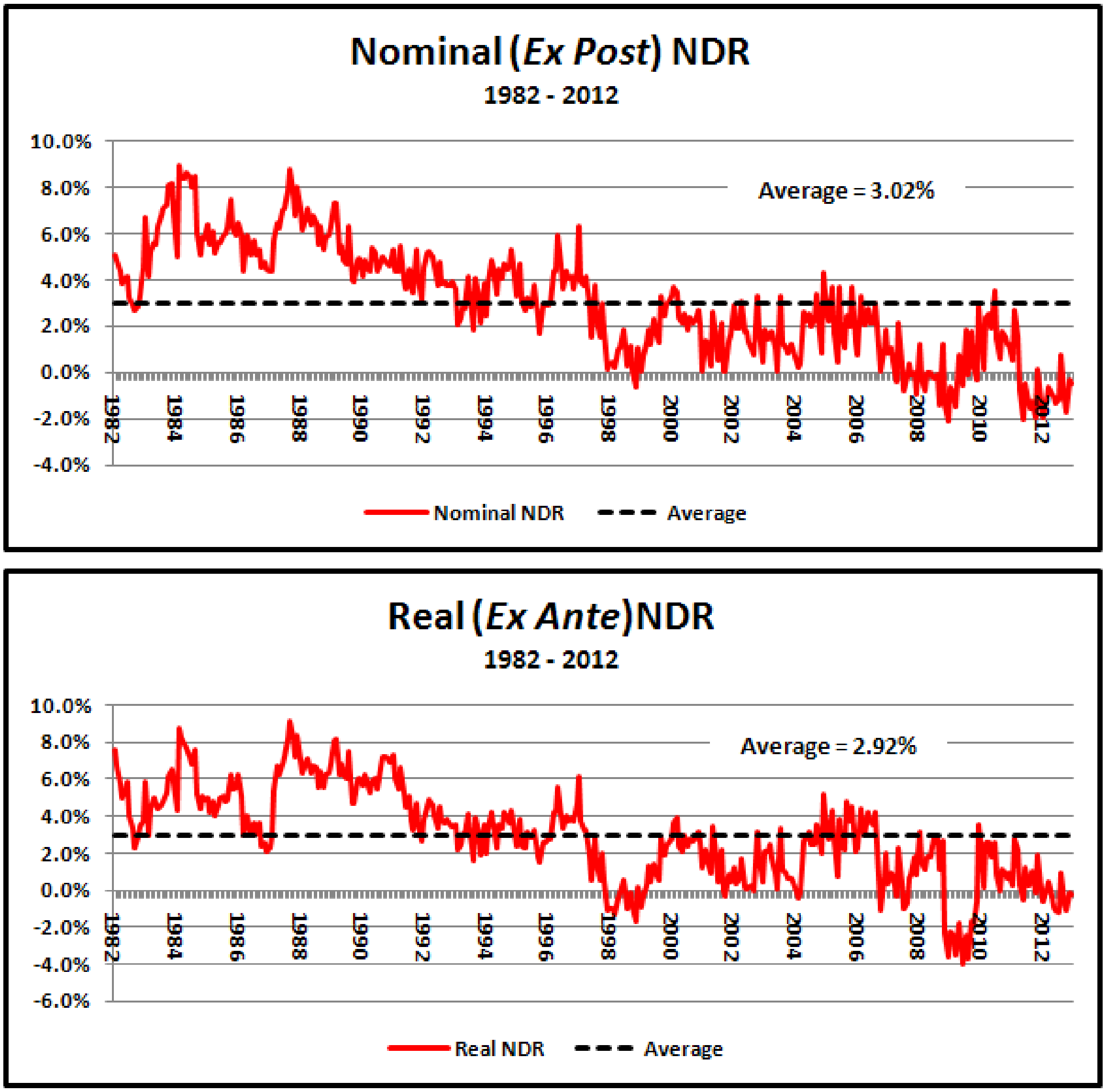


Figure 3

Nominal Net Discount Rate
(Monthly Data)

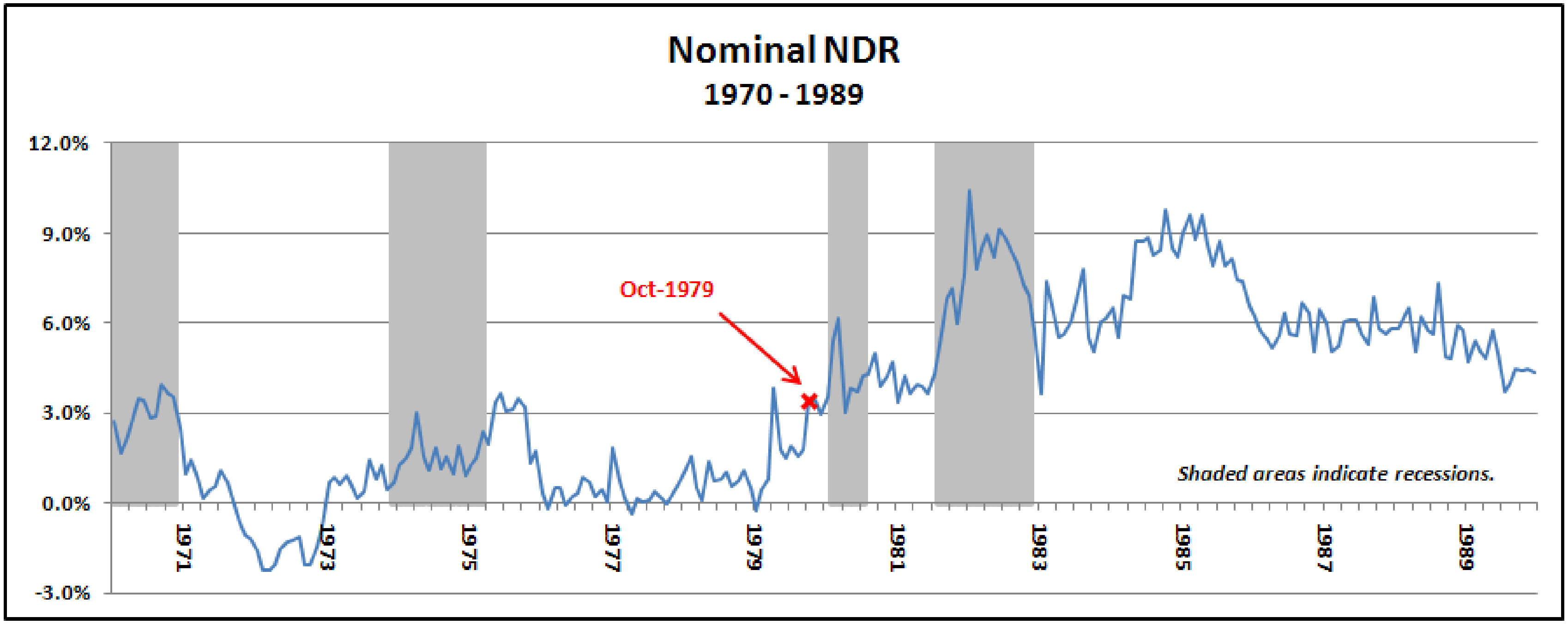
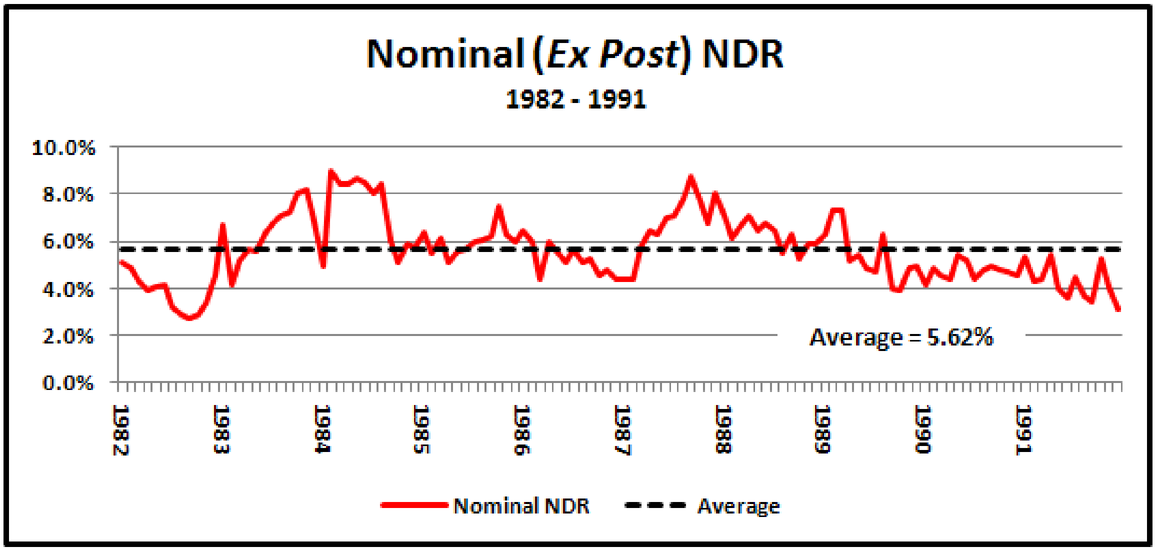


Figure 4

Nominal (NDR_1) and Real (NDR_2) Net Discount Rates

(Monthly Data)



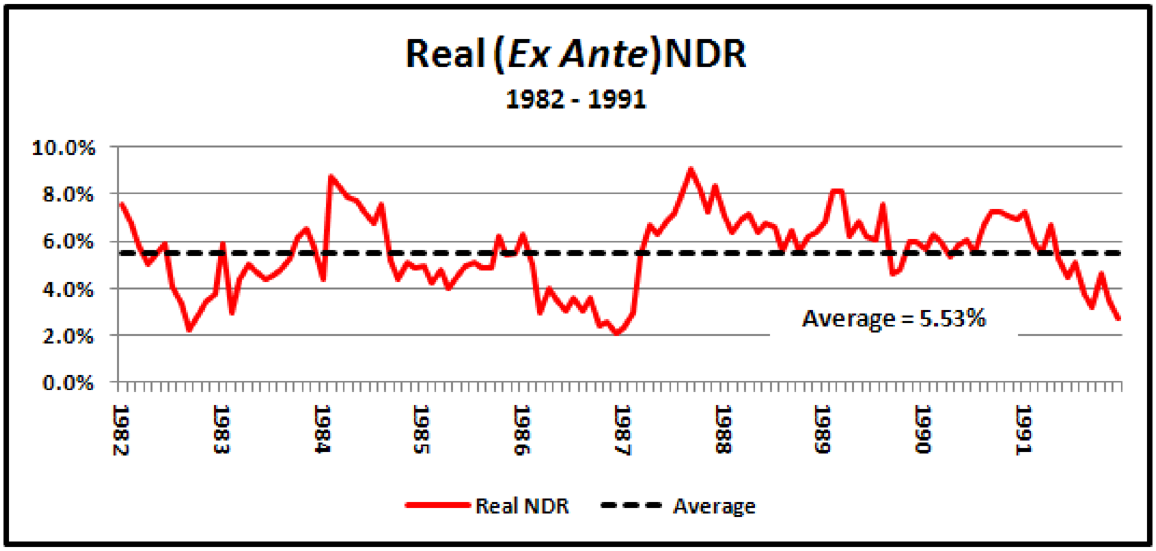
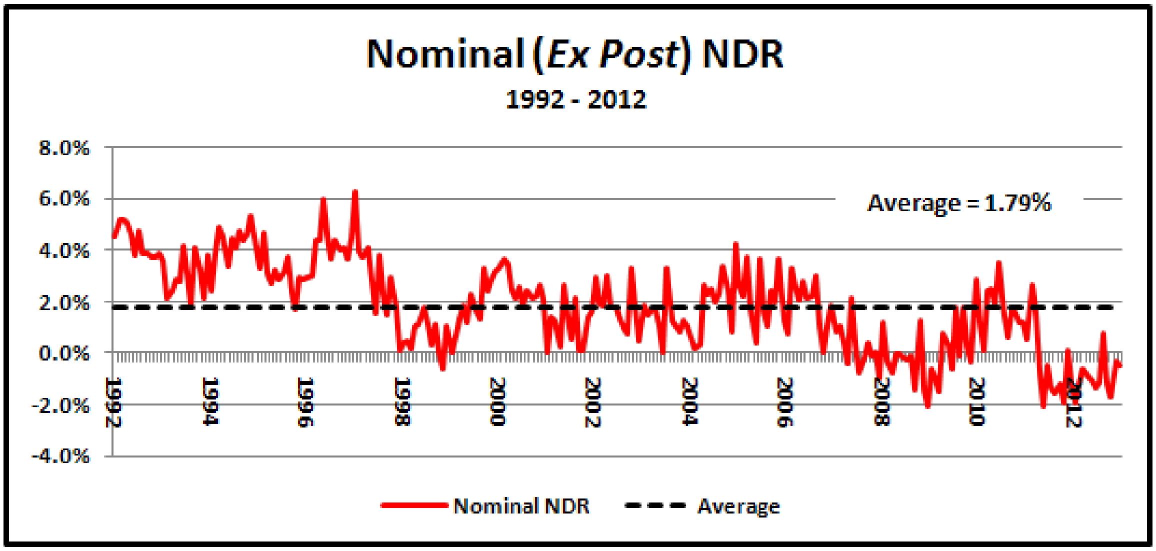


Figure 5

Nominal (NDR_1) and Real (NDR_2) Net Discount Rates
(Monthly Data)



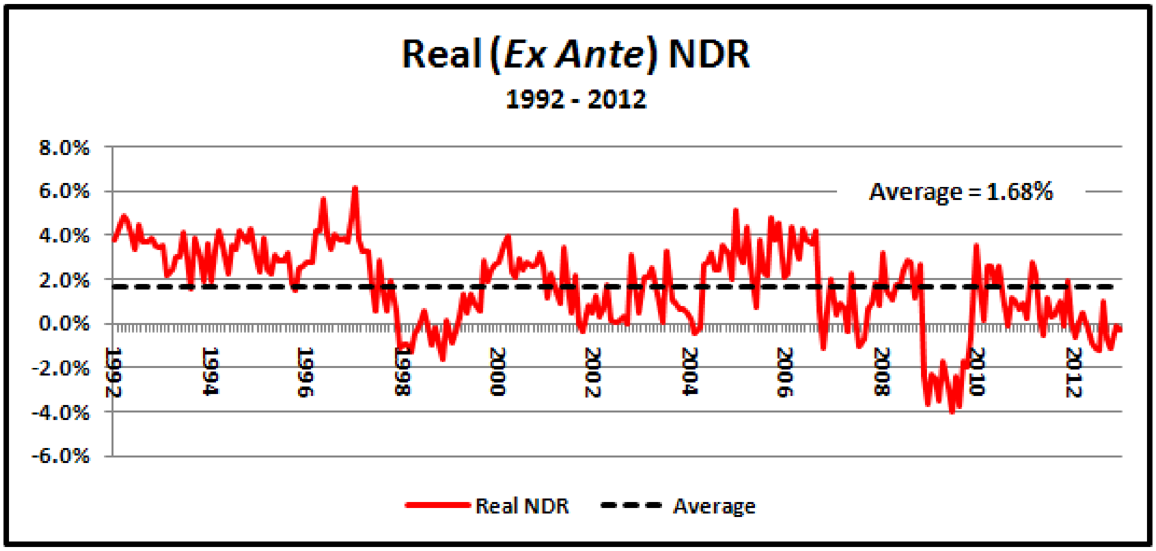
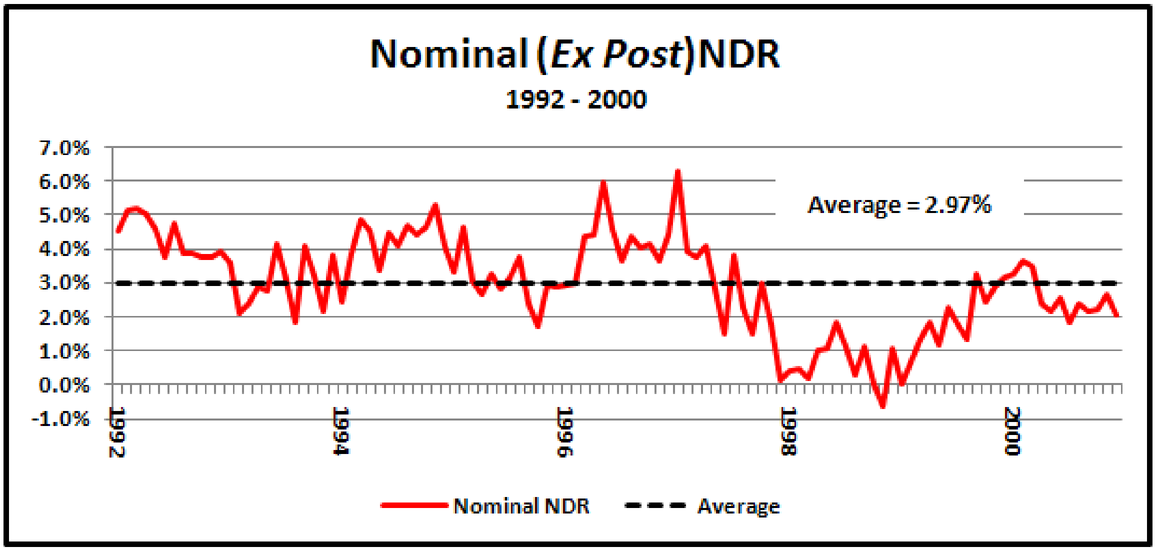


Figure 6

Nominal (NDR_1) and Real (NDR_2) Net Discount Rates

(Monthly Data)



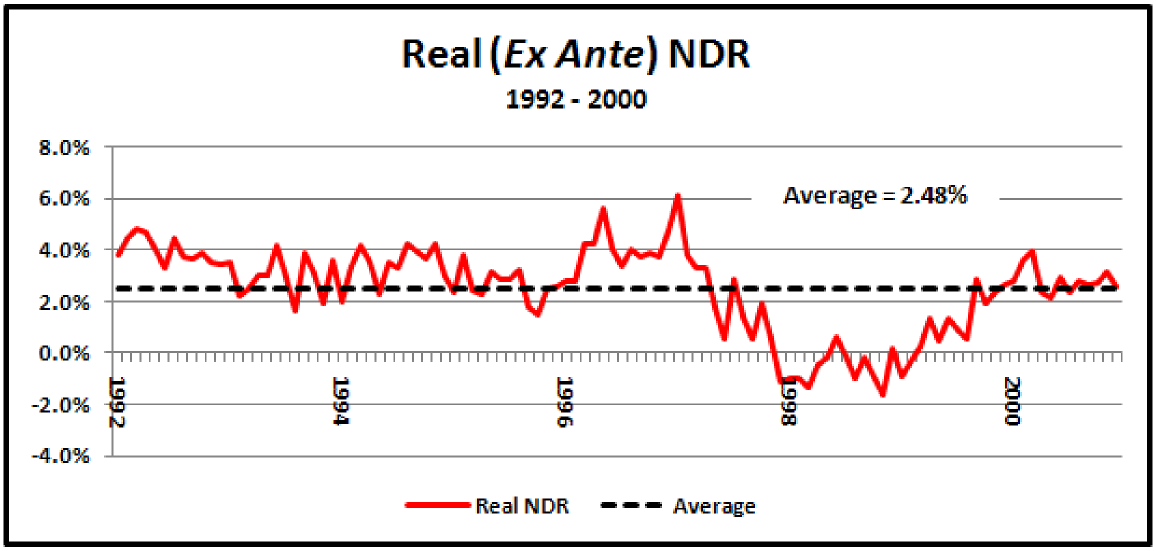
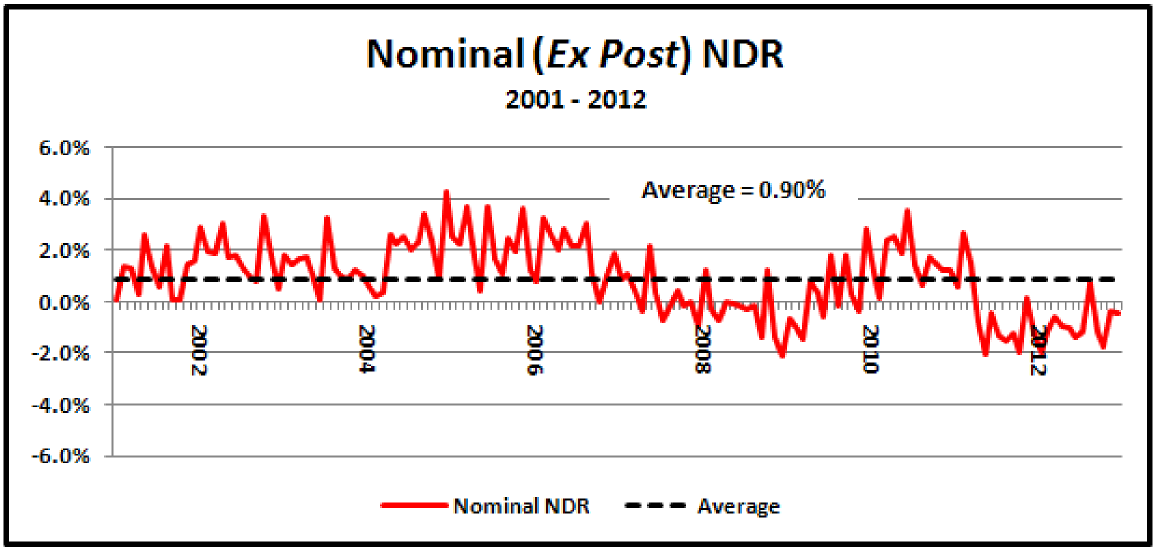


Figure 7

Nominal (NDR_1) and Real (NDR_2) Net Discount Rates

(Monthly Data)



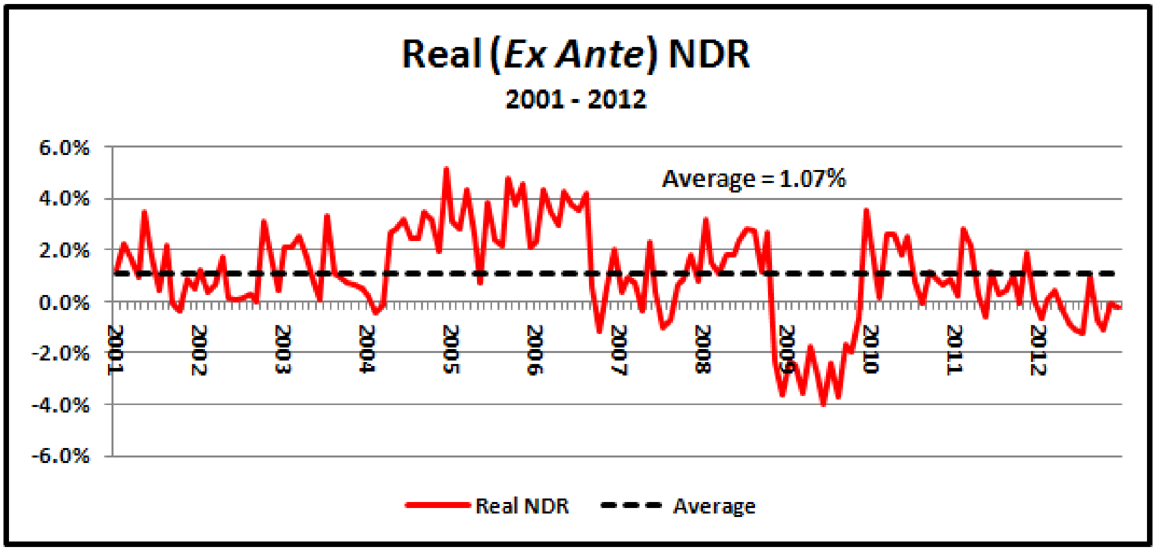


Figure 8

Sample Correlograms:
Nominal (NDR_1) and Real (NDR_2) Net Discount Rates

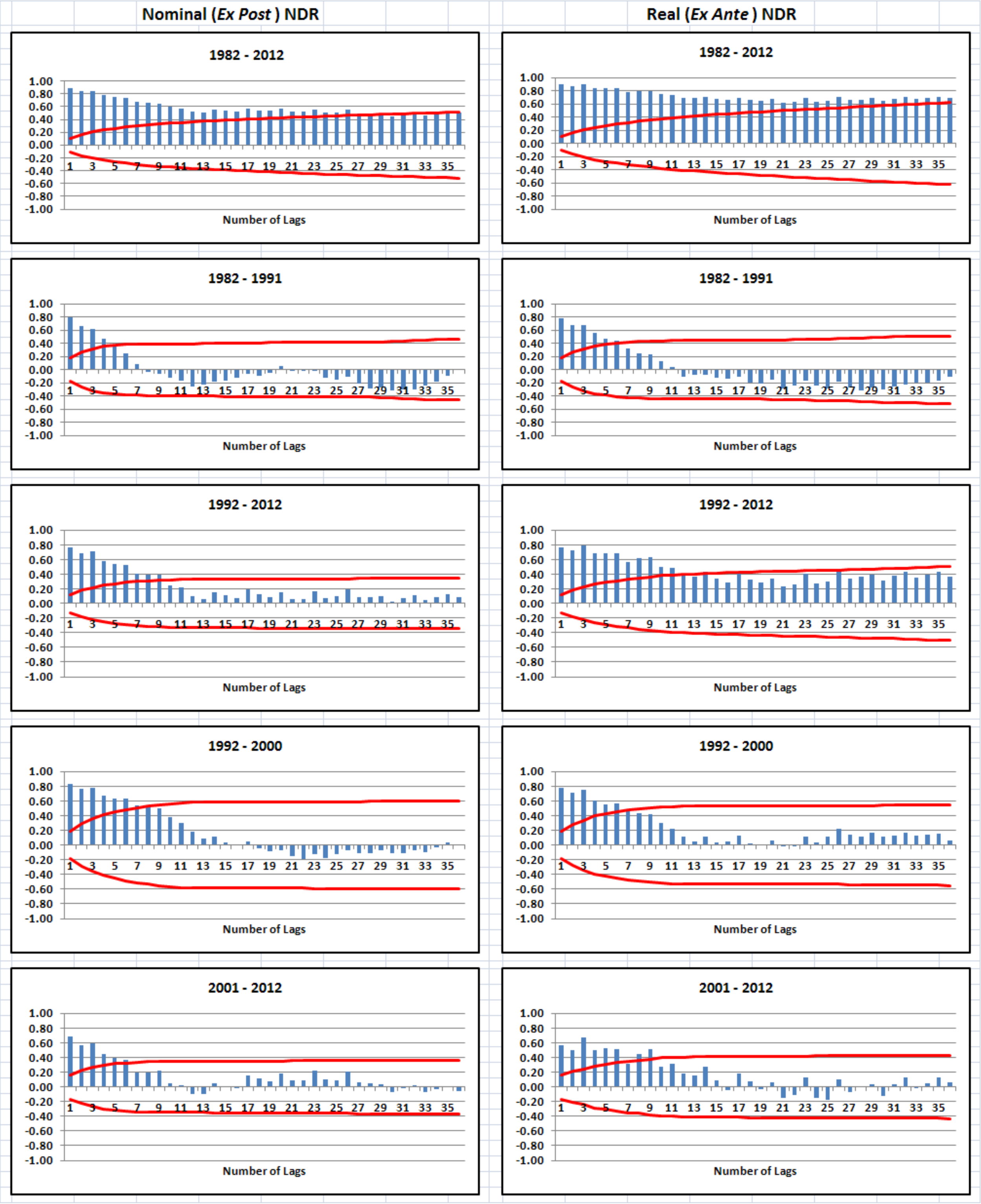


Table 4 - Summary of Unit Root Test Results

	One-Sided p-Values			
		Phillips Peron		
Time Period	ADF	Bartlett Kernel	AR OLS	KPSS
	Nominal (Ex Post) Net Discount Rate			
1982 - 2012	0.64803	0.00031	0.30227	Reject at 1% level
1982 - 1991	0.29939	0.01359	0.17194	CNR at 10% level
1992 - 2012	0.34179	0.00000	0.00390	Reject at 1% level
1992 - 2000	0.14127	0.01312	0.06473	Reject at 5% level
2001 - 2012	0.09160	0.00000	0.00000	Reject at 5% level
	Real (Ex Ante) Net Discount Rate			
1982 - 2012	0.47129	0.00241	0.08015	Reject at 1% level
1982 - 1991	0.00742	0.00593	0.00742	CNR at 10% level
1992 - 2012	0.12838	0.00001	0.00240	Reject at 1% level
1992 - 2000	0.20781	0.05805	0.14361	Reject at 5% level
2001 - 2012	0.09325	0.00002	0.00326	Reject at 10% level

“CNR” = “Can Not Reject”

Table 5 – ADF and PP Unit Root Test Results

	Nominal (*Ex Post*) Net Discount Rate	**Real (*Ex Ante*) Net Discount Rate**
ADF		
	Not stationary for 1982-2012 period.	Not stationary for 1982-2012 period.
	Not stationary for 1982-1991 period.	Stationary for 1982-1991 period.
	Not stationary for 1992-2012 period.	Very weak support for stationarity for 1992-2012 period.
	Very weak support for stationarity for 1992-2000 period.	Not stationary for 1992-2000 period.
	Weak support for stationarity for 2001-2012 period.	Weak support for stationarity for 2001-2012 period.
PP - Bartlett Kernel		
	Stationary for 1982-2012 period.	Stationary for 1982-2012 period.
	Stationary for 1982-1991 period.	Stationary for 1982-1991 period.
	Stationary for 1992-2012 period.	Stationary for 1992-2012 period.
	Stationary for 1992-2000 period.	Weak support for stationarity for 1992-2000 period.
	Stationary for 2001-2012 period.	Stationary for 2001-2012 period.
PP - AR OLS		
	Not stationary for 1982-2012 period.	Weak support for stationarity for 1982-2012 period.
	Not stationary for 1982-1991 period.	Stationary for 1982-1991 period.
	Stationary for 1992-2012 period.	Stationary for 1992-2012 period.
	Weak support for stationarity for 1992-2000 period.	Very weak support for stationarity for 1992-2000 period.
	Stationary for 2001-2012 period.	Stationary for 2001-2012 period.

Table 6 – OLS Estimates of ρ Corrected for Bias

Time Period	**OLS Estimate of ρ**	**Durbin's h-Statisitc**	**Durbin's h-Statisitc p-Value**	**1st Corrected OLS Estimate of ρ (ρ_{ow})**	**2nd Corrected OLS Estimate of ρ (ρ^*)**	**One-Sided p-Value for ρ_{ow} (H_0: $\lvert\rho\rvert=1$)**
Nominal (*Ex Post*) Net Discount Rate						
1982 - 2012	0.897445	6.6448	0.0000	0.907479	~~0.944355~~	0.0323
1982 - 1991	0.791709	2.1213	0.0169	0.820805	~~0.842041~~	0.0520
1992 - 2012	0.769408	5.3783	0.0000	0.782748	~~0.857550~~	0.0176
1992 - 2000	0.776526	0.7156	0.2371	~~0.808541~~	0.796579	0.0539
2001 - 2012	0.567809	3.2141	0.0007	0.587119	~~0.662367~~	0.0140
Real (*Ex Ante*) Net Discount Rate						
1982 - 2012	0.886539	4.9715	0.0000	0.896484	~~0.927044~~	0.0282
1982 - 1991	0.795510	2.4082	0.0080	0.824705	~~0.851121~~	0.0535
1992 - 2012	0.771905	3.8517	0.0001	0.785274	~~0.837686~~	0.0179
1992 - 2000	0.827218	1.1781	0.1194	~~0.860695~~	0.854222	0.0835
2001 - 2012	0.682803	2.3704	0.0089	0.704577	~~0.748191~~	0.0218

Table 7 - Number of Lags with Autocorrelations Significantly Different from Zero (Based on 95 Percent Confidence Interval)

Time Period	Last Significant Lag	Comment
Nominal (*Ex Post*) Net Discount Rate		
1982 - 2012	29	Not Stationary
1982 - 1991	4	Stationary
1992 - 2012	9	Stationary
1992 - 2000	7	Stationary
2001 - 2012	6	Stationary
Real (*Ex Ante*) Net Discount Rate		
1982 - 2012	36	Not Stationary
1982 - 1991	6	Stationary
1992 - 2012	11 + 14	Stationary
1992 - 2000	6	Stationary
2001 - 2012	6 + 8 9	Stationary

Note: "K + N_1 N_2 . . ." indicates significant autocorrelations at lags 1 through K, followed by signifcant autocorrelations at lags N_1 N_2 . . . .

Table 8 – Overall Summary of Results
(Is the NDR Stationary?)

Time Period	Based on Visual Inspection	Based on ADF Test	Based on PP Test (Bartlett Kernel)	Based on PP Test (AR OLS)	Based on KPSS Test	Based on Autoregressive Coefficient Corrected For Bias	Based on Correlogram
Nominal (*Ex Post*) Net Discount Rate							
1982 - 2012	Not Stationary	Not Stationary	Stationary	Not Stationary	Not Stationary	Stationary	Not Stationary
1982 - 1991	Stationary	Not Stationary	Stationary	Not Stationary	Stationary	Stationary	Stationary
1992 - 2012	Slight Trend	Not Stationary	Stationary	Stationary	Not Stationary	Stationary	Stationary
1992 - 2000	Slight Trend	Very Weak Support	Stationary	Weak Support	Not Stationary	Stationary	Stationary
2001 - 2012	Stationary	Weak Support	Stationary	Stationary	Not Stationary	Stationary	Stationary
Real (*Ex Ante*) Net Discount Rate							
1982 - 2012	Not Stationary	Not Stationary	Stationary	Weak Support	Not Stationary	Stationary	Not Stationary
1982 - 1991	Stationary	Stationary	Stationary	Stationary	Stationary	Stationary	Stationary
1992 - 2012	Less of a Trend	Very Weak Support	Stationary	Stationary	Not Stationary	Stationary	Stationary
1992 - 2000	Less of a Trend	Not Stationary	Weak Support	Very Weak Support	Not Stationary	Stationary	Stationary
2001 - 2012	Stationary	Weak Support	Stationary	Stationary	Weak Support	Stationary	Stationary